Control quality enhancement by fractional order controllers

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Zvyšovanie kvality regulácie regulátormi neceločíselného rádu
Príspevok sa zaoberá regulátormi neceločíselného rádu. Uvádza matematický popis neceločíselných regulátorov a metódy ich návrhu. Kvalita a robustnosť regulátorov neceločíselného rádu je porovnaná s klasickými celočíselnými regulátormi. Pre použitie regulátorov neceločíselného rádu je uvedený príslušný algoritmus.

Kľúčové slová: regulátor neceločíselného rádu, syntéza regulátora, regulovaný systém neceločíselného rádu, analýza kvality riadenia

Introduction

PID controllers belong to the dominating industrial controllers (Leššo, 1997) and therefore there is a continuous effort to improve their quality and robustness. One of the possibilities to improve PID controllers is to use fractional order controllers with non integer derivation and integration parts.

The controlled objects are generally of fractional order, however for many of them, the fractionality is very low. Their integer order description can cause significant differences in the adequacy between the mathematical model and the real system (Dorčák, 1994; Sýkorová,1996). The main reasons for using integer order models were the absence of solution methods for fractional order equations. In the previous time important achievements were obtained (Oldham, 1974; Axtell and Bise, 1990; Podlubný, 1994; Dorčák, 1994) which enable to be taken into account the real order of dynamic systems. For fractional order systems (Outstaloup, 1995), fractional controller CRONE has been developed which is a modified PD^{δ} controller.

In this paper we present a synthesis of fractional $PI^{\lambda}D^{\delta}$ controllers, analysis of their behaviour and simulation methods. We point out the non-adequate approximation of non-integer systems by integer order models and differences in their closed loop behaviour.

Properties of fractional order control system and fractional order controllers

Let's consider a feed-back control system with an unit gain in the feed-back loop (fig.1). Where Gr(p) is the controller transfer function and Gs(p) is the controlled system transfer function.

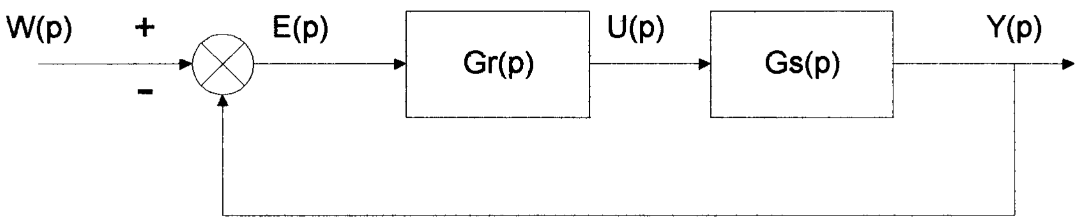

Fig.1. Feed - back control loop.

The fractional order controlled system is represented by the fractional order model with the transfer function (Podlubný, 1994),

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$$G_s(p) = \frac{1}{a_2 p^{\alpha} + a_1 p^{\beta} + a_0} \tag{1}$$

where α and β are generally real numbers ($\alpha > \beta$).

$G_r(p)$ is represented by the fractional PD^{δ} controller with the transfer function (Podlubný, 1994),

$$G_r(p) = K + T_d p^{\delta} \tag{2}$$

or by the fractional $PI^{\lambda}D^{\delta}$ controller with transfer function (Podlubný, 1994),

$$G_r(p) = K + T_i p^{-\lambda} + T_d p^{\delta} \tag{3}$$

where λ is an integral order, δ is a derivation order, K is a proportional gain, T_i is an integration constant and T_d is a derivation constant.

Synthesis of fractional order controllers

For the synthesis of integer PD and PID controllers, different methods are used, e.g. the method of dominant roots, the Naslin's method, the optimal module method, the symmetrical optimum method, the standard form method and the different empirical methods.

Our approach is based on the modification of dominant roots method.

- **Synthesis of fractional PD^{δ} controller**

This controller has about one more parameter δ comparing to the integer PD controller. This gives us one additional degree of freedom , and we can except from desired stability level S_t and dumping level T_l define maximal allowed control static deviation E_t.

The design procedure consists of two parts :

1. Design of parameter K

The proportional parameter K influences the value of static deviation E_t, control time T_r and the overregulation P_r. Generally, with the increased parameter K, the control time T_r decrease and the static deviation E_t is lowering:

$$E_t = \frac{1}{a_0 + K} 100[\%] \tag{4}$$

2. Design of parameters Td, δ

We define the required stability level $S_t=a$ and the dumping level $T_l=b$. These requirements satisfy a couple of conjugate complex roots (poles)

$$p_{1,2} = -a \pm \frac{a}{b} i \tag{5}$$

We use a characteristic equation similar, to that obtained by the classical method of dominant roots. The characteristic equation of the fractional order control loop has the form

$$G_r(p) G_s + 1 = 0 \tag{6}$$

After substituing of the fractional order controller transfer function (2) and the fractional order controlled system transfer function (1) and after some corrections we obtain the characteristic equation in the following form

$$a_2 p^{\alpha} + a_1 p^{\beta} + T_d p^{\delta} + \left(a_0 + K\right) = 0 \tag{7}$$

- **Synthesis of fractional PI$^{\lambda}$D$^{\delta}$ controller**
  The design procedure is similar to that for the design of fractional PD$^{\delta}$ controller :

1. Design of parameter K

For the determination of the parameter *K* for the real time the same procedure as by PD$^{\delta}$ controller can be used.

2. Design of parameters Ti, Td, λ, δ

The closed loop characteristic equation has the form (5). After substituing of the controller and the controlled system transfer function into equation (5) and after modifying, the characteristic equation has the following form

$$a_2 p^{\alpha} + a_1 p^{\beta} + T_d p^{\delta} + T_i p^{-\lambda} + \left(a_0 + K\right) = 0 \tag{8}$$

Algorithm for the fractional order controllers

The control algorithm was designed according to the control scheme shown in Fig.1. This algorithm consists of the following steps:

1. A difference (e) between the desired (w) and the output (y) value determination

$$e(t) = w(t) - y(t) \tag{9}$$

or in the discrete form :

$$e_m = w_m - y_m \tag{10}$$

for the discrete time step (m=1,2,…).

2. Control determination

The control value *u* can be determined from (3) by the inverse Laplace transformation

$$u(t) = Ke(t) + T_i e^{(-\lambda)}(t) + T_d e^{(\delta)}(t) \tag{11}$$

For discrete time control can be expressed in the form

$$u_m = Ke_m + T_i h^{\lambda} \sum_{j=0}^{m} q_j e_{m-j} + T_d h^{-\delta} \sum_{j=0}^{m} d_j e_{m-j} \tag{12}$$

where *h* is the time step. For the approximation of the fractional derivation and integral we use equation after (Dorčák, 1994). The binomial coefficients d_j and q_j were calculated from the generally recurrent equation

$$a_j = \left(1 - \frac{1+\alpha}{j}\right) a_{j-1} \tag{13}$$

where $a_0 = 1$ and α is a derivation or integral order.

The numerical algorithm requires store the whole history. For improving their effectiveness we have used the "short memory" principle (Dorčák, 1994). Besides the "short memory", the control quality is influenced by the time step *h*. The maximal and minimal control value have to be taken into account because of the limitations of their sources (e.g. gas input).

Fractional order controllers can be realised as a software or passive or active electrical elements.

Comparison of the fractional and integer order controllers

Example

Here, the fractional $PI^{\lambda}D^{\delta}$ controller is compared with the standard controller designed for the required steady deviation $E_t<1.3\%$. For the real time $K=75$ was determined from (4). After substituing into (8) instead of *p* use one of the complex conjugate roots (5) and the calculated value of K, we obtain the following equation

$$0.8(-2+5i)^{2.2} + 0.5(-2+5i)^{0.9} + T_d(-2+5i)^{\delta} + T_i(-2+5i)^{-\lambda} + (1+75) = 0 \tag{14}$$

This algebraic equation has four unknown parameters and cannot be unambigously solved. For an unambigous solution, we have chosen two parameters e.g. T_i, λ. We have used a weak integrator with order $\lambda=0.5$ and integration constant $T_i=315$. After substituing into (14) we obtain the equation for the determination of T_d and δ. The designed parameters are :

$$K = 75, T_i = 315, T_d = 25.878, \lambda = 0.5, \delta = 1.205 \tag{15}$$

The fractional order controller, in comparison to the integer order controller, has the lower control surface by 111%, the overregulation by 93%, the control time by 107% and the steady control deviation by 135%.

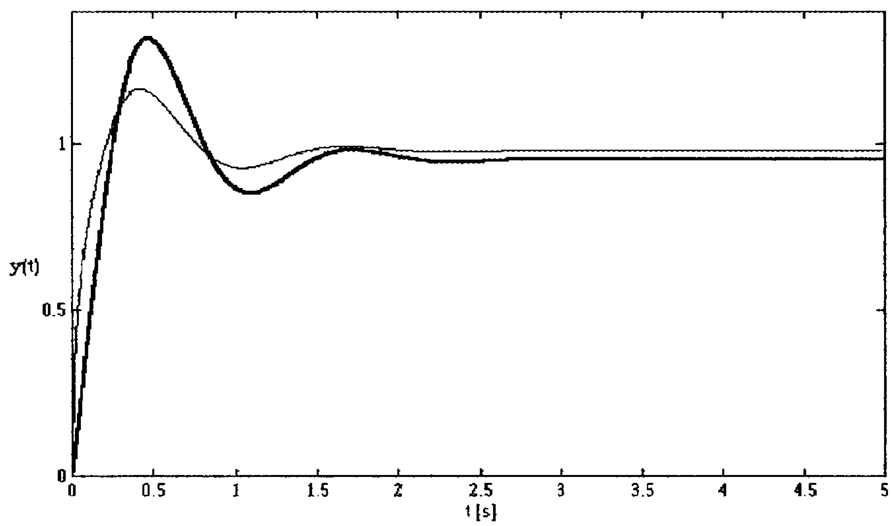

Fig.2 : Comparison of fractional and integer PID controller.

Fig.2 represents the transient characteristics of the feed-back fractional (slim line) and the integer (thick line) PID controllers for the integer controlled system.

Robustness improvement

Robust controller is less sensitive to the parameter changes of controlled system. The uncertainty can be caused by the non-precise identification. The fractional order controllers are less sensitive to changes of controlled system parameters. As an example, the parameter a_1 was changed from 0.5 to -10. To Fig.3 is the transient function of the integer PID controller and in Fig.4 is the transient function of the fractional $PI^{\lambda}D^{\delta}$ controller. As we can see the integer controller is behind its stability level but the fractional $PI^{\lambda}D^{\delta}$ is still stable. The same is valid for the controller parameters variations.

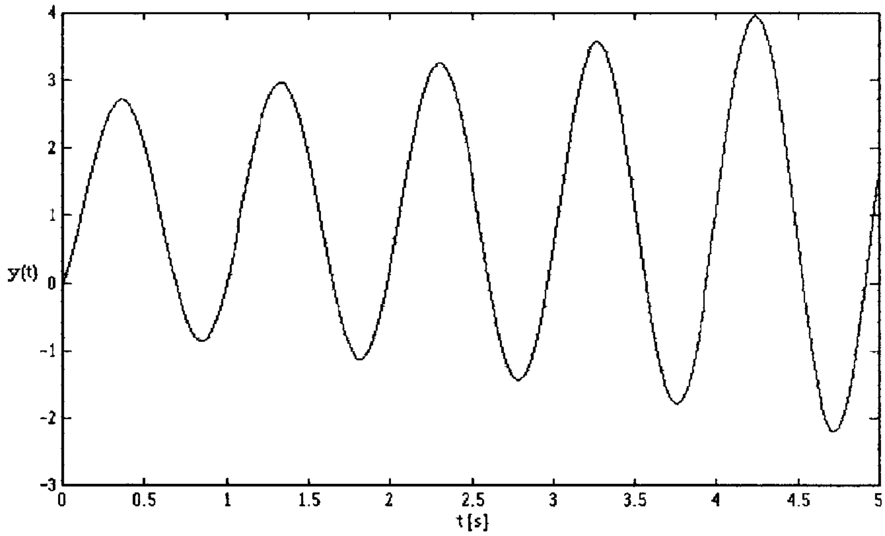

Fig.3 : Transient function of integer PID controller.

From this result and from the results of (Dorčák,1994; Sýkorová et al.,1996) it follows that the non-integer PID controllers are more robust than the integer controllers.

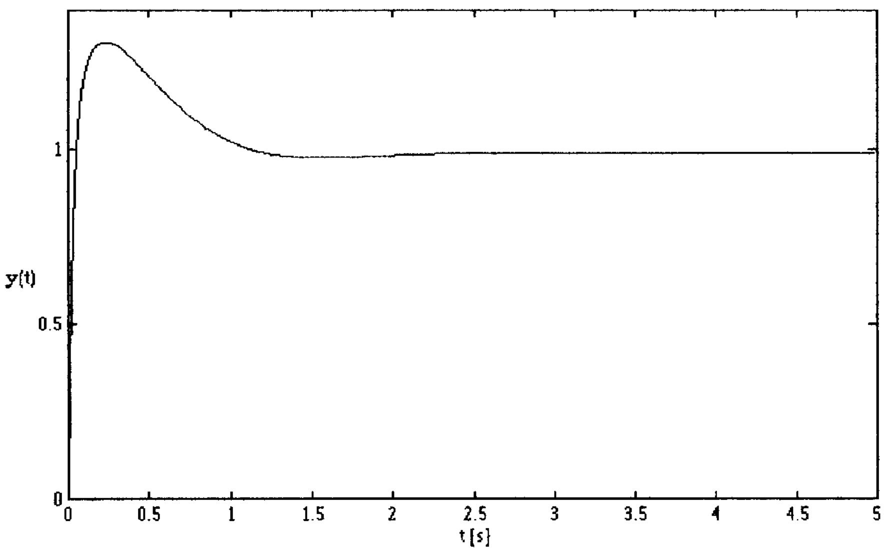

Fig.4 : Transient function of fractional $PI^{\lambda}D^{\delta}$ controller.

Conclusion

The outlined design method of fractional order controllers enables them to be used for the integer and non-integer order controlled systems (Petráš et al., 1997). The fractional order controllers are designed in the frequency domain for the determined stability and dumping level. They can significantly improve static and dynamic control system properties. The fractional order controllers are less sensitive to controlled systems and controllers parameters variations and can be used as robust controllers.

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