Identification of nonlinear systems based on mathematical – physical analysis and least square method

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Identifikácia nelineárnych systémov založená na matematicko-fyzikálnej analýze a metóde najmenších štvorcov

The paper deals with the identification of nonlinear systems described by the nonlinear difference equations of the certain type. These equations are known in literature dealing with the systems modelling by means of neural network as the models of the second type. In some cases, when these models are nonlinear also in parameters and we want to use the least square method for parameter estimation, it is necessary to transform the models into linear forms. Our attention has chiefly been paid to the practical utilization of the proposed algorithm and to this verification on simulated example.

Key words: *nonlinear system, identification, recursive least square method.*

Introduction

One of the principal conditions for successful process control is deep knowledge of the process properties. We need to known the steady state and dynamic behaviour of the controlled plant. Characteristics of disturbances entering a system has to be find, the possibility of the time-invariant behaviour of a system has to be studied and a number of other information are necessary for the right choice of the optimal control strategy.

The above mentioned information can be obtained by the thorough mathematical-physical analysis (MFA) of the plant. Results of the MFA are models in the form of linear or nonlinear differential or difference equations. The basic problem of this approach is the parameter estimation of these equations. The parameters are functions of the physical and technological quantities, which are not often at disposal e.g. mass and heat transfer coefficients, mixture properties etc. The optimal solution of the problem is then the combination of MFA and the experimental system identification (ESI). Unknown coefficients of the model equations can be estimated by the ESI even in the cases, when the MFA output is only the model structure in the form of differential or difference equations.

If we succeed in the formulation of a nonlinear model that is linear in parameters, the recursive least square method (RLSM) can be used for parameter estimation. The on-line identification and consecutive adaptive control can then be realized for such cases.

Suppose that the results of the MFA are in the form of the difference equation of the special type (nonlinear in variables but linear in parameters). In case of the equation nonlinear in parameters, the equation can be transformed into the model linear in parameter and the RLSM can then be used.

Description of the certain nonlinear plant type

Our attention will be further focused on the models, described by the nonlinear difference equation (Narendra and Parthasarathy, 1990)

$$y(k) + f_s[\alpha_j, y(k-1), y(k-2), \ldots, y(k-n)] - \sum_{j-1}^{m} b_j u(k-j) \tag{1}$$

where: *u(k), y(k)* represents the input/output pair of discrete signals of a SISO system, $f_s[.]$ is a nonlinear function of the variable y and linear or nonlinear function in parameters, α_j are coefficients (*j* – their sequence number) of the nonlinear part and b_j are coefficients of the linear part of the model.

Example of the nonlinear plant models of the second type (see Narendra and Parthasarathy, 1990)

$$y(k) + \frac{\alpha_1 y(k-1)}{1+\alpha_2 y^2(k-1)} = bu(k-1) \tag{2}$$

Suppose that (2) is a result of MFA and the analysed plant is at disposal for experimental identification. For such case, the values of the model coefficients need not be necessary known.

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The parameter estimation of the nonlinear plant with RLSM

Using the RLSM for the parameter estimation needs the model linear in parameters. The parameters are designated as γ_j. Their estimations in k = 0, 1, …, N are designated $\hat{\gamma}(k)$. These parameters are changed to reduce the difference between the plant output *y(k)* and the predictive output $\hat{y}(k)$. The aim of the procedure (EIS) is to minimize the squared error over *N* patterns

$$E = \sum_{k=0}^{N} \left(y(k) - \hat{y}(k) \right)^2 \tag{3}$$

The identification accuracy is given through the mid-squared error S=E/(N-1).

The recursive least square method

RLSM is given in the form of three equations (RLSM is derived e.g. in Drábek, Macháček, 1987, 1992)

$$v(k) = P(k) f(k+1) \left(e_x(k) + f^T(k+1) P(k) f(k+1) \right) \tag{4a}$$

$$\hat{\gamma}(k+1) = \hat{\gamma}(k) v(k) \left(y(k+1) - f^T(k+1) \hat{\gamma}(k) \right) \tag{4b}$$

$$P(k+1) = \frac{1}{e_x(k)} \left[P(k) - v(k) f^T(k+1) P(k) \right] \tag{4c}$$

where: *v(k)* - auxiliary vector defined in order to simplify the equation (4b) *P(k)*, - up-dated matrix *p* x *p* where *p* is a number of the parameters, $e_x(k)$ - factor of the dynamical exponential forgetting, *f(k+1)* - data vector; his elements are the variables of the transformed model, $\hat{\gamma}(k)$ - parameter estimation vector, *y(k+1)* - the discrete value of the response in *k+1*.

The algorithm starts with zero values of the parameters $\hat{\gamma}(0)$. Equation (4c) serves for the actualization of the *P(k)* matrix. The initial matrix

$$P(0)=K\,I \tag{5}$$

where: *I* is the unit matrix and *K* is constant determined in the course of simulation.

The transformation of the model

Model (2) of the plant is nonlinear model in parameters. The transformation of Eq. (2) is necessary. We go out from the idea that the model nonlinear in parameters can be transformed into linear one with help of the polynomial *M[y(k)]*. The more polynomial elements, the better accuracy of the approximation. In our case we obtain

$$M[y(k-1)]=-y(k-1)+y^3(k-1)-y^5(k-1)+ \ldots \tag{6}$$

Substituting the right side of this equation in Eq. (1), we obtain model in the form

$$Y(k)-\gamma_1 y(k-1)+\gamma_2 y^3(k-1)-\gamma_3 y^5(k-1)+\ldots=bu(k-1) \tag{7}$$

The data vector

$$F(k)=[-y(k-1)-y^3(k-1)-y^5(k-1)-\ldots+u(k-1)]^T \tag{8}$$

and the parameter vector

$$\hat{\gamma}(k) = \left[\hat{\gamma}_1(k) \quad \hat{\gamma}_2(k) \quad \hat{\gamma}_3(k) \quad \ldots. \quad \hat{b}(k) \right]^T \tag{9}$$

The quality of the above mentioned model is checked-up within the framework of the simulation process. Algorithm of RLSM had been programmed in MATLAB and the simulation results are presented in the following chapter.

Simulation of the identification

The difference between the models quality with different number of the data vector elements is perceptible from Fig.1. In this figure the parameter change of α_l in sixty step from α_l =1 to α_l=0,8 is presented. For the models with three elements -see $y_{M3}(k)$ course and with nine elements – see $y_{M9}(k)$ course. The input step change signal is *u(k)*=0,25; 0,5. The faster adaptability with nine elements is evident.

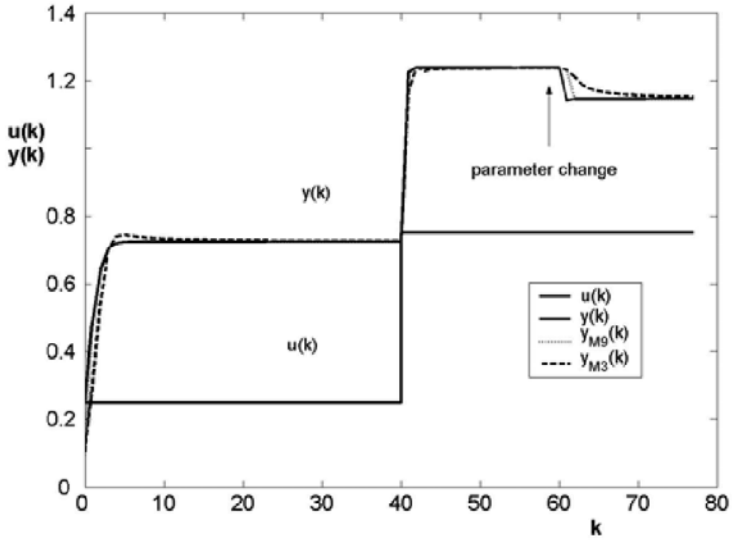

Fig.1. The influence of the parameter change on the proces quality.

Conclusion

It follows from the simulation results that the identification of the above mentioned plant is satisfactory. The described method can be used in such case when the model structure in the form of the difference equation is known and the experimental identi– fication is possible. These two pro– cedures need to be finished by the simulation calculations, the result of which is suitable value of K.

It is very well known there does not exist an universal method of the nonlinear problems solve. This paper is a contribution to extend of „status-quo". We suppose that the research will continue in the area of the control synthesis.

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