Influence of discretization method on the digital control system performance

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Vplyv diskretizačnej metódy na výkonnosť digitálnych riadiacich systémov

The design of control system can be divided into two steps. First the process or plant have to be convert into mathematical model form, so that its behavior can be analyzed. Then an appropriate controller have to be design in order to get the desired response of the controlled system. In the continuous time domain the system is represented by differential equations. Replacing a continuous system into discrete time form is always an approximation of the continuous system. The different discretization methods give different digital controller performance. The methods presented on the paper are Step Invariant or Zero Order Hold (ZOH) Method, Matched Pole-Zero Method, Backward difference Method and Bilinear transformation. The above mentioned discretization methods are used in developing PI position controller of a dc motor. The motor model was converted by the ZOH method. The performances of the different methods are compared and the results are presented.

Key words: *Continuous and Discrete-Time Model, Discretization, ZOH, Bilinear Transformation, Sampling Time.*

Introduction.

Modern controllers are realized by microprocessor based digital circuits, or process-control computers. These devices can be characterized by discrete operation, where the control algorithms are implemented by computer program. The main parameter of these systems is the sampling time. Due to the sampling, these systems are called discrete-time systems. Additionally, since the processors have infinite word length, the signals are discretized in amplitude, and are encoded. In contrast to the controller, generally the controlled process is analogue. Therefore the plant is connected to the controller by A/D and D/A converters. Two basic ways can be used for controller design. The first strategy is based on the analogue system model, the controller design is carried out on the continuous time domain. The second method uses discrete system model and the design procedure is done on the discrete time domain. In both cases the system have to be transferred to discrete time system. The methods influence the system performance, as it is shown in the followings.

Analogue system model

The D/A converter converts the digital control signal into analogue and provides continuous signal on the process input. Usually D/A converters realize zero order hold function (in the followings ZOH), where the signal values between two constitutive sampling are equal to the last sampling values. In contrast to the D/A converter, the A/D converter converters the analogue signal to digital input quantity. There are two approximation possibilities for digital controller modeling, as continuous and discrete model. In the first case the system is considered as continuous system. The sampling interval introduces a delay, that is presented by the transfer function W_{ds}.

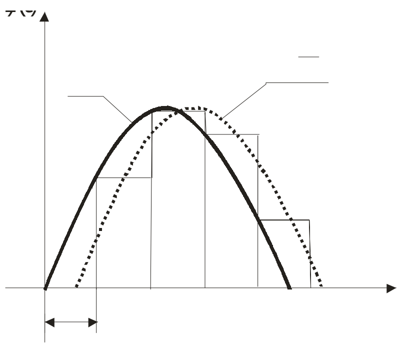

Fig.1. Approximation of sampling and hold by Dead time.

Dead time does not influence the amplitude-frequency characteristic, but the phase is increased. The phase margin increase is:

$$\Delta\varphi = -\frac{\omega_c T_s}{2}\left[rad\right]$$

The time delay, $T_s/2$ is taken into consideration by the transfer function $W_d(s)$:

$$W_d(s) = e^{-s\frac{T_s}{2}}$$

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(Recenzovaná a revidovaná verzia dodaná 19.11.2003)

Design steps based on the continuous model

a. $W_c(s)$ have to be determined for $W_p(s)$ continuous plant

b. System behaviour have to be checked based on the frequency characteristic of the opened loop transfer function: $W_0 = W_c W_p$

c. Based on the maximum phase margin increase - $\Delta\varphi$, at the natural frequency ω_c, the sampling time can be calculated from the form:

$$\omega_c \frac{T_s}{2} = \frac{\pi}{180\Delta\varphi}$$

If $W_c(s)$ and T_s is known, the digital controller algorithm can be simple realized.

Discrete system model

The hybrid system can also be approximated by discrete model. The background of this approximation is, that the controller is a discrete system, and from the controller point of view the plant also looks discrete. The discrete model is shown in Fig. 2. The main disadvantages of the method are, that neither the sampling time nor the system behaviour are know when the continuous model is converted into discrete one. The $W_{pd}(s)$ plant model contains a ZOH on the input side.

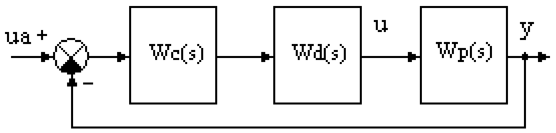

Fig.2. Continuous model.

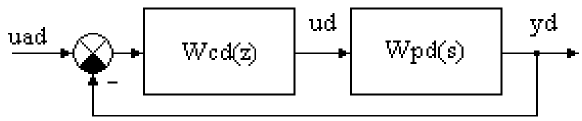

Fig.3. Discrete model.

Design steps based on the continuous model

a. Selection of T_s sampling time based on the parameters of $W_p(s)$. T_s depends on the time constants of the system. One practical approximation should be:

$$\frac{1\sum^{m} T_k + T_h}{25} < T_s < \frac{25}{12}\frac{\sum_{1}^{m} T_k + T_h}{25}$$

where: T_k presents time constant, and T_h is the dead time.

b. Determination of the discrete W_{pd} plant model of the continuous W_p system.

c. Selection of the discrete time algorithm. The controller with $W_{cd}(z)$ transfer function is chosen. PI, PD, PIPD are the most frequently used strategies. Other strategies are: Deadbeat, optimal control, adaptive control, State-Space model.

d. Determination of the K_{cd} gain constant of the controller. K_{cd} determines the φ phase margin at the W_c frequency.

Discretization methods for analogue systems.

Different techniques are used to convert continuous systems into discrete systems. However, it is to be noted, that the continuous system can only be approximated and the discrete system can never be exactly equivalent. Different methods can result different controller performances. The most important methods are introduced in the following part.

Step invariant or Zero Order Hold (ZOH) method

The step response of the discrete system is the same as the continuous system at the sampling instants. It is supposed that the system is predicted by a ZOH. The method is used for conversion plants into discrete time domain:

$$H(z) = (1 - z^{-1}) Z\left[L^{-1}\frac{H(s)}{s}\right]$$

Ramp invariant method

The step input is replaced by a ramp signal. The method is also called First Order Hold (FOH) method, and can be used for transformation of continuous controllers.

$$H(z) = \left[\frac{(1-z^{-1})^2}{Tz^{-1}}\right] Z\left[L^{-1}\frac{H(s)}{s^2}\right]$$

Matched pole-zero method

The poles of the s domain are mapped directly into the z domain. The relationship between s and z is: $z = e^{sT}$

Backward difference method

The derivative of the function is replaced by the difference between the samples:

$$\frac{dy}{dt} = \frac{y(n) - y(n-1)}{T};$$

$$s = \frac{1 - z^{-1}}{T}$$

Bilinear transformation

The method is also called the Tustin transformation, or trapezoid approximation. The transformation equation is:

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

The bilinear method is the most commonly used method for convert controllers into the discrete time domain..

Position controller with dc servo motor

The different discretization methods are demonstrated in this paragraph. The servo motor is given by the next transfer function:

$$W_m = \frac{52.1}{s(1.21s+1)}$$

If T_s=0.001s, and the ZOH method is used, the plant discrete time model is given by:

$$W_m(z) = \frac{0.0264z + 0.0263}{z^2 - 1.989z + 0.9889}$$

The PID controller continuous form is given by:

$$W_c = \frac{0.525s^2 + 5.022s + 4.4}{0.005s^2 + s}$$

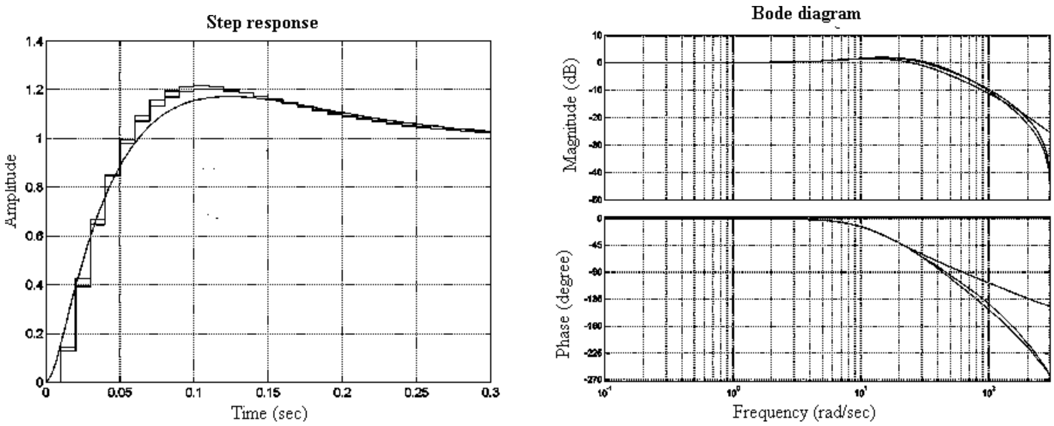

Fig.4. Step responses and Bode diagrams of different discrete systems.

The controller's discontinuous form using Tustin approximation is:

$$Wc(z) = \frac{55.02z^2 - 105z + 50}{z^2 - z}$$

The pole-zero mapping method was also tested. The performances of the methods were compared by Matlab simulation. The Bode diagrams and the step responses are:

Characteristics of the different discretization methods

ZOH or step invariant method. Delay caused by the ZOH is the disadvantages of the method. It introduces phase lag and distorts the frequency response of the controller. Therefore it is generally used for plant discretization.

Ramp invariant method. Gives good results and may be used when converting continuous controllers.

Matched Pole-Zero. In this technique**, the poles of the s domain** The gain of the two systems is matched at a critical frequency, by choosing an arbitrary gain constant. Aliaising effects are not taken into consideration.

Backward difference method. Stable continuous systems results always stable digital equivalents. The jω axes in the s plain does not map to the unit-circle in the z plain, thus degrading the frequency response. Higher sampling frequency gives a better approximation.

Bilinear transformation or Tustin transformation. This is the most commonly used method. The system in the z domain is always stable, if the continuous-time controller is stable.

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