Determining of Signal Derivatives in Identification Problems - FIR Differential Filters

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The paper presents a methodology for designing regression filters. The signal x needs to be locally approximated in the neighbourhood of the instant t using a polynomial of an appropriate degree. The values of the signal derivatives are then determined basing on the polynomial parameters. An advantage of the regression differentiating filters is that it is possible to obtain a filtered signal with the same length as that of the original signal. An example of solving the parameter identification problem in case of robot with four degrees of freedom has been also presented.

Keywords: *identification; differential filters; low-pass filter*

Introduction

Identification method and its generalizations using the object inverse model require information on time derivatives of the input and output signals (Janecki and Cedro 2007, Cedro and Janecki 2005). The required derivative order depends on the order of differential equations describing the object. In reality the signal derivatives can be rarely obtained directly. They usually have to be determined on the base of the registered signal. At present only computer techniques are used for system identification and control. This means that we do not operate on registered analog signals, but only on the samples taken in regularly-spaced time intervals, called the sampling period. A problem of evaluation of signal derivatives only on the base of accessible signal samples arises (Soderstrom and Stoica 1994, Janecki 1995).

The problem of signal derivatives determining is a subject of many papers. A modification of known algorithms of signal derivatives determining is proposed in (Mocak et al. 2007). Application of the Fourier Transform together with smoothing of the discrete signal is proposed by the authors of (Jun-Sheng and Zu-Xun 1996). Pintelon and Schoukens worked, among others, on using of the IIR and FIR digital filters for signal differentiation and integration in (Pintelon and Schoukens 1990).

Differentiating and integrating systems have found their applications - among others - in automatic control systems, signal analyzers, analog-to-digital and digital-to-analog converters (Rabiner and Gold 1975).

However authors of many publications notice a significant problem that the measurement noise causes difficulties in exact determining of an appropriate signal with the differentiation and integration methods. The digitization process besides sampling comprises also signal quantization, connected for example with analog-to-digital converters application. It is known that the quantization noise has a uniform distribution of probability density. The measured signal can be also distorted by a noise of other reason.

The designed filters determining appropriate derivatives should have the characteristics as close to the ideal as possible but also should significantly eliminate the measurement noise. Authors of the paper propose then application of the designed FIR filters for determining of appropriate order derivatives, with simultaneous elimination of measurement noise and keeping the signal length the same as before the filtration.

Problem Statement

Consider a dynamic object described by ordinary differential equations

$$f(y^{(n)}(t),\ ...,\dot{y}(t),y(t),u^{(n-1)}(t),\ ...,\dot{u}(t),u(t),\theta)=0 \tag{1}$$

where y and u is object output and input, respectively, f is a certain function (linear or nonlinear) and θ represents the unknown object parameters. Let's assume that we possess sample trains of input and output signal

$$y(\Delta k),u(\Delta k),\quad k=1,2,...,K. \tag{2}$$

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(Review and revised version 15. 09. 2011)

Our goal is parameter identification of θ on the base of the registered samples. The identification procedure can be based on minimisation of the coefficient

$$J(\hat{\theta}) = \sum_{k=1}^{K} \in (\Delta k)^2, \tag{3}$$

where $\in (\Delta k)$ is appropriately defined model error and $\hat{\theta}$ is the parameter estimation. For example we can assume that

$$\begin{aligned} \in (\Delta k) = f(&\hat{y}_n(\Delta k),\ ...,\hat{y}_1(\Delta k),\hat{y}_0(\Delta k),\\ &\hat{u}_{n-1}(\Delta k),\ ...,\hat{u}_1(\Delta k),\hat{u}_0(\Delta k),\hat{\theta}), \end{aligned} \tag{4}$$

where generally $\hat{x}_i(\Delta k)$ denotes the estimation of the derivative $x^{(i)}(t)$ of signal $x(t)$ at the instant $t = \Delta k$. The algorithm of derivative estimation determining will be called the differential filter. Notice that because of the measurement noise resulting – among others – from quantization noise, the differential filters should have a low-pass character, thus they should determine the signal derivatives in the range of low frequencies and assure appropriate suppression in the range of high frequencies (Lyons 1999).

Regressive Differential Filters

An approach to filter design using local regressive signal analysis based on polynomials of appropriate order will be presented in this point. Regressive filtration is based on local approximation of the signal $x$in the neighbourhood of the instant $t$with the use of an appropriate order polynomial and then on determining the values of the filtered waveform on the base of determined polynomial parameters.

Consider the discrete case where the continuous signal $x(t)$is sampled with the sampling period Δ. Consecutive signal samples will be denoted by $x_k = x(\Delta k),\ k \in Z$. For every instant Δk we approximate the value train $x(\Delta m)$ in the neighbourhood of Δk by the polynomial of the form

$$p_{m-k} = a_{0k} + a_{1k}\Delta(m-k) + ... + \frac{1}{n!}a_{nk}\Delta^n(m-k)^n. \tag{5}$$

Because the approximation is to be of a local character, we will use a certain weighting function $w(\Delta k)$ characterising the size of the area where the approximation is performed and in the consequence the cut-off frequencies and the transmission characteristics of the obtained filters. Function $w$may fulfil the condition $w(\Delta k) = w_k = 0$ for $\Delta k > T$ for a certain T value or equivalently $k > M$, where $M = T/\Delta$. In this case the approximation interval length will be equal to $2M+1$. Thus at every instant the polynomial parameters (as a function of $k \in [0, K-1]$) will be determined by minimisation of the goal function

$$J(a_{0k},...,a_{nk}) = \frac{1}{2}\sum_{m=0}^{K-1}(x_m - p_{m-k})^2 w_{m-k} \tag{6}$$

We obtain from the above that the parameter vector

$$\theta(\Delta k) = \theta_k = [a_{0k}...a_{nk}]^T \tag{7}$$

fulfils the conditions

$$A_k\,\theta_k = b_k, \tag{8}$$

where A is a matrix of elements

$$A_{i,j,k} = \sum_{m=0}^{K-1}\frac{1}{i!j!}(\Delta(m-k))^{i+j}w_{m-k}, \tag{9}$$

where $i = 0,...,n,\ j = 0,...,n$ and b is a vector of elements

$$b_{ik} = \sum_{m=0}^{K-1}\frac{1}{i!}x_m(\Delta(m-k))^i w_{m-k},\ i = 0,...,n. \tag{10}$$

To determine the filter properties we assume that the signal $x(\Delta k)$ is determined for every k, then when determining the Fourier Transform of the equation, defined by

$$X(\omega) \approx F[x(\Delta k)] = \sum_{k=-\infty}^{\infty} x_k e^{-i\Omega k} = \sum_{k=-\infty}^{\infty} x(\Delta k)e^{-i\omega\Delta k} \tag{11}$$

we obtain

$$A\,\theta(\omega) = b(\omega). \tag{12}$$

Elements of vector $b(\omega) = F[b_k]$ are equal to

$$\begin{aligned} F[b_{ik}] &= \tfrac{1}{i!}F\left[\textstyle\sum_{m=-\infty}^{\infty} x_{k+m}(\Delta m)^i w_m\right] = \\ &= \tfrac{1}{i!}F\left[\textstyle\sum_{m=-\infty}^{\infty} x_{k-m}(-\Delta m)^i w_{-m}\right] = \\ &= \tfrac{1}{i!}X(\omega)F\left[(-\Delta m)^i w_{-m}\right] = X(\omega)W_i(-\omega) \end{aligned} \tag{13}$$

where

$$W_i(\omega) = \frac{1}{n!}F[(\Delta k)^i w_k] = \frac{1}{n!}i^i\frac{\partial^i W(\omega)}{\partial \omega^i}, \tag{14}$$
$$W(\omega) = W_0(\omega) = F[w(\Delta k)].$$

Finally

$$\begin{aligned} &A\,\theta(\omega) = W(\omega)X(\omega), \\ &W(\omega) = [W_0(-\omega)...W_n(-\omega)]^T. \end{aligned} \tag{15}$$

The transmittance between $\theta(\Delta k)$ and the signal $x(\Delta k)$is equal to

$$A^{-1}W(\omega) \tag{16}$$

In particular case, when $n = 0$ we get

$$a_0(\Delta k) = \frac{\sum_{m=-\infty}^{\infty} x_m w_{k-m}}{\sum_{m=-\infty}^{\infty} w_m}. \tag{17}$$

In this case the value $a_0(\Delta k)$ is equal to the value of the signal $x(\Delta k)$ filtered by the filter of impulse response equal to $h(\Delta k) = w(\Delta k)/\sum_{m=-\infty}^{\infty} w(\Delta m)$. The value $a_1(\Delta k)$is, in turn, the evaluation of the slope of the signal $x(t)$at the instant $t = \Delta k$, and in general $a_i(\Delta k),\ i = 0,1,...,n$ is the approximation, in the low frequency range, of the response of the ideal differential filter with transmittance $(i\omega)^i$.

The algorithm determining the expressions $y_i(\Delta k) = a_i(\Delta k)$ for $i = 0,1,...,n$ will be called the discrete differential filter. Finally the transmittance vector of the discrete differential filter is determined from

$$G(\omega) = A^{-1}W(\omega). \tag{18}$$

At last notice that the ideal differential element of order i has the transmittance $(i\omega)^i$ because $\frac{d}{dt}e^{i\omega t} = (i\omega)^i e^{i\omega t}$. So in the low frequency range following condition should be fulfilled

$$G(\omega) \approx [1\ i\omega...(i\omega)^n]^T. \tag{19}$$

In case of regressive filters the weighting function has been defined as the Harris

$$w(t) = 0,36 + 0,49\cos(\pi t/M) + 0,14\cos(2t\pi/M) + 0,01\cos(3t\pi/M) \tag{20}$$

and rectangular window

$$w(t) = \frac{1}{2M+1} \tag{21}$$

for $t \in [-M, M]$ and $w(t) = 0$ otherwise.

We will compare the characteristics of differential filters and ideal differential elements. We can see in Fig. 1 that the approximation is good in the low frequency range. For the frequency $\omega = 10[rad]$ and $M = 100$ the average approximation error of individual filters is respectively equal to: 0.0533604%, 0.0442875%, 3.11234%, 2.84098%.

Differentiaiton Results

A quality of the derivatives determined with the use of elaborated differential filters will be examined in this point. Consider the second order derivation operation on the sinusoidal signal

$$x(t) = \sin\omega t \tag{22}$$

with $\omega = 10$. Assume that the signal is sampled with the sampling period $\Delta = 0.001$ $[s]$ and that the analog to digital conversion is performed by the 16-bit A/D converter of the range $[-1,\ 1]$.

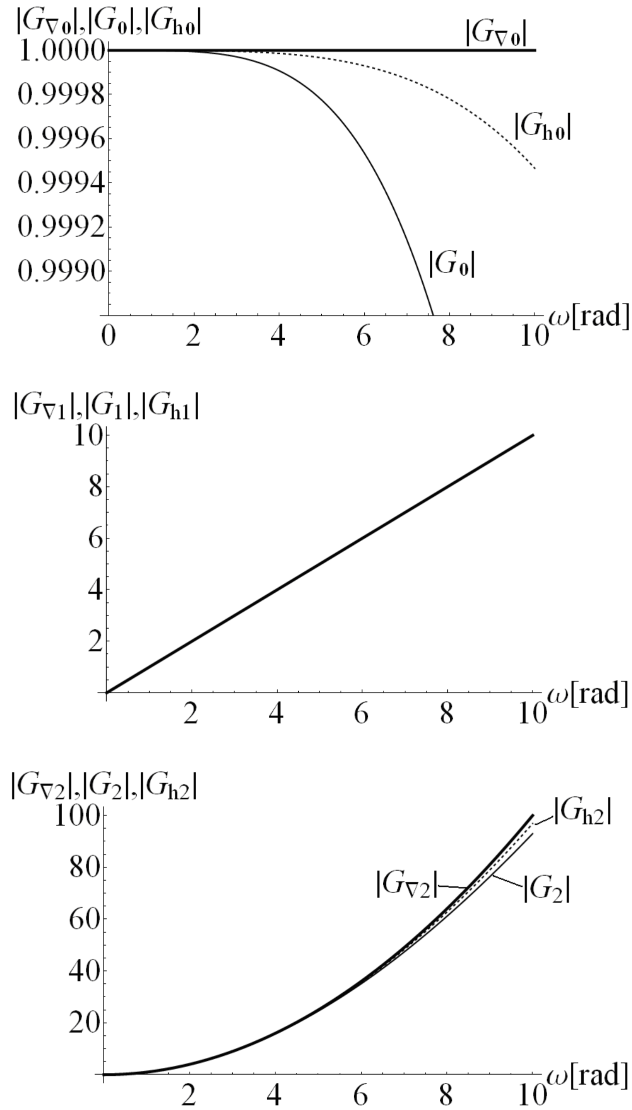

Fig. 1: Characteristics of ideal $|G_{\nabla i}|$ and regressive differential filters with Harris$|G_{hi}|$and rectangular$|G_i|$window for $i = 0,1,2$.

As a result of performing the differentiation with the use of difference quotient, the calibration error is equal to 11%. The error determined for elaborated filters has been significantly reduced. For the considered regressive differential filter the error does not exceed 0.02%. The Harris window ensures good approximation of the filter characteristic within the transmission band and thereby distorts the useful components of the signal in a minor degree. It should be remembered that the relative differentiation error of the quantized signals strongly depends on the signal frequency and grows with lessening the frequency.

Assuming that sampling period is constant, the differentiation accuracy can be enhanced by the appropriate weighting function and the filter width M.

The advantage of elaborated filters is obtaining of signals with the same length as before the filtration. The adverse effect of signal shortening often occurs at designing the FIR filters. The presented filters not only keep the signal length but also allow for removing of the high-frequency components of the signal, for example the noise. In Fig. 2 we can observe in a large zoom the beginning of the signal $\ddot{x}(t)$ before and after the filtration, where we can see no shortening of the filtered signal and significant noise elimination.

Identification of the Manipulator Model

To demonstrate the advantages of the elaborated filters we will consider strongly nonlinear object of many parameters being the subject of identification. An example of such an object will be a manipulator with four rotational joints of the structure presented in Fig. 3 (Craig 1995, Spong and Vidyasagar 1997). Let's introduce following notation: let

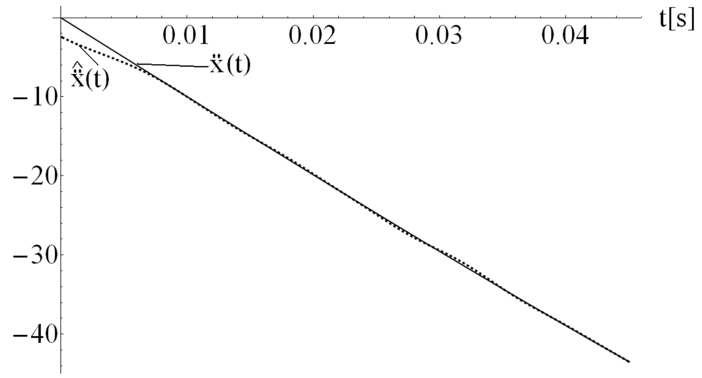

Fig. 2: Boundary effect after using the regressive filters for the signal with noise.

$\varphi = [\varphi_1\ \varphi_2\ \varphi_3\ \varphi_4]$ denotes a vector of joint variables functioning as generalized coordinates, m_o - the mass and l_o - the length of element o. We assume for simplification that the elements are perfectly stiff and the mass distribution is straight, where particular masses are concentrated at the end of each element.

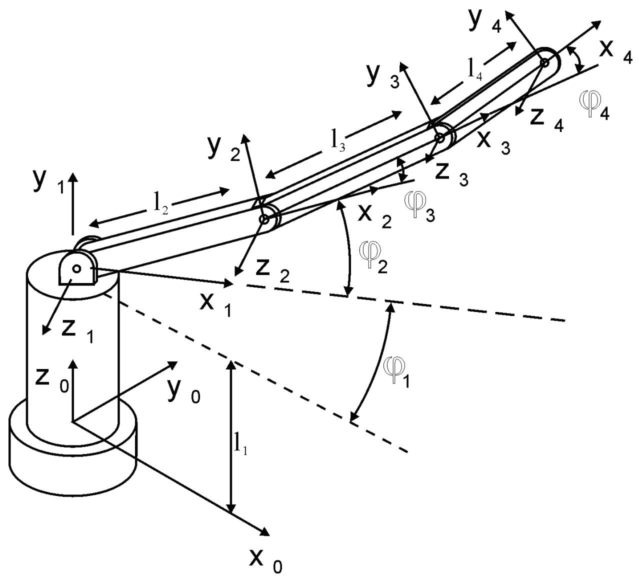

Fig. 3: The manipulator model.

Using the expressions for kinetic and potential energy, K and Π, respectively, we obtain the Lagrange equations of the second kind

$$\frac{d}{dt}\frac{\partial K_{ov}}{\partial \dot{q}_j} - \frac{\partial K_{ov}}{\partial q_j} + \frac{\partial \Pi_{ov}}{\partial q_j} = Mzr_j, \tag{23}$$

$$Mzr_j = K_{pj}(\varphi_{rj} - \varphi_j) - K_{dj}\dot{\varphi}_j, \tag{24}$$

where $o = 1,2,3,4,\ j = 1,2,3,4$ and K_p, K_d are regulator parameters and φ_r- set signals.

On the base of the Lagrange equations of the second kind the equations of the robot dynamics have been determined in a symbolic manner.

In consecutive points a simulation of the robot system in the closed system with the PD regulators has been carried out. An inverse problem method for robot parameters identification on the base of measurement data has been implemented and the identification results have been presented.

Simulation

Simulation results of robot equations in a closed system with PD regulators have been presented in this point. The simulation results will be further used as input data for identification algorithms. In the beginning we define the set signal $\varphi_r = [\varphi_{r1}\ \varphi_{r2}\ \varphi_{r3}\ \varphi_{r4}]$. We assume that the signal is a step function, appropriately delayed (with different delay for every part). The function has been additionally filtered by a first order low-pass filter of the limit frequency $\omega_g = 0.025\,[\mathrm{rad}]$.

The waveforms of variables in the closed system have been presented in diagram Fig. 4. It should be remembered that our goal is to generate signals for the purposes of identification process. In such a case it is more reasonable to select such set signals and regulator parameters that the obtained signals carry much information on the object.

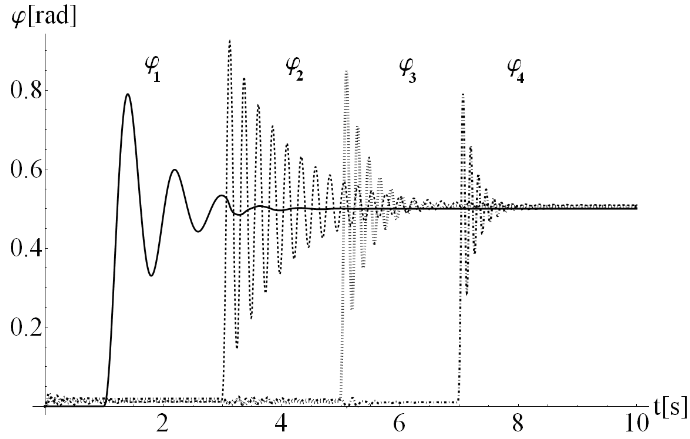

Fig. 4: Waveforms of variables φ_1, φ_2, φ_3, φ_4.

Identification

For identification of robot system parameters $\theta = [m_1, m_2, m_3, m_4, l_1, l_2, l_3, l_4]$ a method, whose schematic diagram is presented in Fig. 5, has been used. It is assumed that the measurements of generalised variables φtrajectory and appropriate input signals Mzr are available. On the base of current estimation of object parameters $\hat{\theta} = [\hat{m}_1, \hat{m}_2, \hat{m}_3, \hat{m}_4, \hat{l}_1, \hat{l}_2, \hat{l}_3, \hat{l}_4]$ the estimation of input signals $Mz\hat{r}^f$ is determined.

The equations have the same structure as equation Eq. 23, where in position of unknown parameters θ the estimations $\hat{\theta}$ have been used, and in position of generalised variables φ and their derivatives $\dot{\varphi}$, $\ddot{\varphi}$ (which are not measured) - estimations of those variables φ^f, $\dot{\varphi}^f$, $\ddot{\varphi}^f$, obtained with the use of appropriate differential filters. The identification problem is to determine the parameter estimations minimising the quality factor

$$J(\hat{\theta}) = \frac{1}{T}\int_0^T (Mz\hat{r}^f - Mzr^f)^2 dt, \tag{25}$$

where Mzr^f is the filtered input signal. The advantage of the described method, versus the classic output error method, where the object output signal φ and its estimation $\hat{\varphi}$ are compared, is no need to solve a series of differential equations in each iteration of the algorithm minimising the quality factor. This allows speeding up the identification procedure significantly.

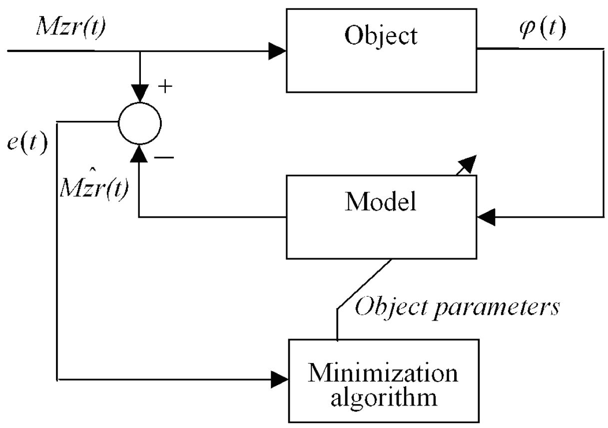

Fig. 5: Identification diagram.

Deliberate significant limitation of the set signal spectrum should be noticed here. Thanks to this, despite the fact that the robot system is strongly nonlinear, following relationship is fulfilled for the filtered signals

$$Mz\hat{r}^f \cong Mzr^f \, for \; \hat{\theta} = \theta. \tag{26}$$

Determine the values of particular signals and their derivatives, necessary in the identification process.

To reduce the computational cost we will use a discrete equivalent of a quality factor Eq. 25 for identification.

The obtained estimates $\hat{\theta}$ =[46.6791, 25.0535, 14.8852, 5.0470, 0.4096, 0.8005, 0.6011, 0.1992] differ from real parameter values θ =[50, 25, 15, 5, 0.4, 0.8, 0.6, 0.2] in a small degree. Slight differences are the effect of system nonlinearity, thus equation Eq. 26 is fulfilled only with a certain approximation.

Identification and Measurement Noise

In this point we will examine how far the elaborated regressive filters eliminate the measurement and quantization noise. We will also examine the influence of the measurement and quantization noise on the result of identification process with the use of finite elements differentiation method and elaborated filters.

The signal processing theory comprises activities aimed on selection of substantial information on the examined phenomena and elimination of redundant information. It is commonly known that the measured signals contain components resulting from the disturbances. In our case the quantization noise value is connected directly with the number of bits of the 16-bit A/D converter. The total value of the measurement noise will be determined by the sum of random and quantization noise.

Using the same identification method and elaborated filters following parameters have been obtained for the noisy signal $\hat{\theta}$ =[46.311, 25.0531, 14.8745, 5.0558, 0.4111, 0.8005, 0.6011, 0.1991].

Using the finite elements method following parameters have been obtained for the noisy signal $\hat{\theta}$ =[47.7121, 24.6767, 15.1828, 5.08051, 0.00819064, 0.799134, 0.595849, 0.200085].

Comparing the obtained results we can state that the differential filters eliminate the measurement noise in a major degree and the parameters determined in the identification process are close to the actual ones. Traditional differentiation does not ensure noise elimination and the identified parameters differ significantly from the actual ones.

Using the elaborated filters in identification methods we obtain well determined parameters in case of quantization on the level of 16-bit cards.

Conclusions

Elaborated differential filters have low-pass character. This feature enables removing of high-frequency components of the signal, for example the noise. The cut-off frequency depends mainly on the approximation section length and the form of weighting function. Elaborated differential filters ensure determining of appropriate derivatives of signal with errors far more less than simple differentiation methods, what plays particularly important role in the identification process.

The advantage of the regressive differentiation filters if the possibility of obtaining the filtered signal of the same length as before the filtration. This enables to use this type of filters in the on-line identification.

The designed algorithm enables to obtain a series of differential filters of orders up to n-th, where n is the order of the approximating polynomial. Thanks to this, as a result of single launch of the algorithm, n consecutive derivatives of the signal being filtered can be obtained, what significantly shortens the calculation time.

Filtration of the signals with extremely different waveforms requires the analysis of signal amplitude characteristics and selection of appropriate set of parameters every time.

On the base of the performed calculations it has been noticed that the general approximation quality of the ideal differential filters is falling with the rise of the filter order.

During the identification of nonlinear systems with the inverse method a necessity of input signal spectrum limitation arose, what has been performed with the use of a low-pass filter. Limitation of the input signal spectrum is of a critical importance in nonlinear system identification.

The applied identification method does not require solving of differential equations but only determining of appropriate derivatives. In various calculations which have been performed, proper operation of the method for more complicated mechanical systems and for systems of greater number of identified parameters has been stated.

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