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Robust Decentralized Controller Design for Large Scale Systems: Subsystems Approach

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The paper addresses the problem of the robust output feedback PI controller design for complex large-scale systems with state output decentralized structure. The proposed design method is based on the Generalized Geršgorin Theorem and the V-K iteration method to design robust PI controller guaranteeing feasible performance achieved in subsystems level and for the full system. The proposed method excludes limit of system order in LMI solution, while PI controller design procedure is feasible on the subsystem level. Finally, numerical example is given to illustrate the effectiveness of the proposed method.

Keywords: *decentralized controller, generalized Geršgorin theorem, V-K iteration method, PI controller, LMI, BMI.*

Introduction

The theory of complex or large-scale systems studies how relationships between subsystems give rise to the collective behaviors of a whole system interacts and forms relationship with its environments. Nowadays, most industrial processes are naturally belong to the class of large-scale systems which need control strategy using the system approach. One of the main problems of large-scale system is high dimension which restricts possibility to synthesize controller using LMI/BMI approach. Our experience shows that, in the case of output feedback design procedure, LMI can be feasible if matrix dimensions of LMI formulation is about fifty and it will be worse in the case of BMI. In this paper, our attention is to study a suitable control strategy to overcome this obstacle.

In this paper, we focus on the control of linear large-scale dynamic systems using decentralized approach. We assume that, large-scale system consists subsystems with order as small as possible to design a robust decentralized controller by BMI. The stability and robustness properties of complex system are checked in LMI framework. Recently, in the frequency domain, the so-call Equivalent Subsystem method has been developed for robust decentralized controller design in [Kozáková,Veselý and Osudký (2009)]. The main idea of this approach is that, the controller is designed on the subsystems level for predefined performance and also guarantee stability and performance for complex system. In this paper, we pursue the same idea. In the state space, the robust controller is designed on the subsystem level using BMI approach and then the robustness and performance of complex system is checked using V-K iteration procedure [El Ghaoui and Balakrishnan (1994)] in LMI.

The robust decentralized PI controller is designed using the polytopic description of the uncertain system and applying the robust optimal control design procedure with extended cost function as in [Rosinová and Veselý (2006)] for state-space subsystems generated in each vertex of the polytopic uncertainty domain.

In this paper, the proposed controller design procedure is not proved, but we hope that, it can give interesting results in many practical situations. The paper is organized as follows: Section 2 includes preliminaries and problem statement, Section 3 presents the main results followed by a simple example in Section 4. Conclusions are drawn in the last section.

Notation: Matrices, if not explicitly stated, are assumed to have compatible dimensions. I denotes the identity matrix of corresponding dimensions.

Problem Formulation and Preliminaries

Consider the following linear large-scale stable system with polytopic uncertainty described

$$\begin{aligned} \dot{x}(t) &= A(\xi)x(t) + B(\xi)u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $y(t) \in R^l$ is the output (measured output). The matrices $A(\xi), B(\xi) \in S$ belong to a polytope S with N vertices $S_1, S_2, \ldots, S_N$ which can be formally defined as:

$$S := \left\{ A(\xi) \in R^{n\times n}, B(\xi) \in R^{n\times m} : A(\xi) = \sum_{i=1}^{N} \xi_i A_k,, B(\xi) = \sum_{i=1}^{N} \xi_i B_k, \sum_{k=1}^{N} \xi_k = 1, \xi_k \geq 0 \right\} \tag{2}$$

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Matrices A_k, B_k can be split in two parts

$$\begin{aligned} A_k &= A_{dk} + A_{mk} \\ B_k &= B_{dk} + B_{mk} \\ k &= \{1, 2...N\} \end{aligned} \tag{3}$$

where $A_{dk} = diag\{A^k_{jj}\}, B_{dk} = diag\{B^k_{jj}\}, j = \{1, 2..., M\}$ - block diagonal matrices corresponding to subsystems and matrices A_{mk}, B_{mk} are diagonal off matrices, that is $A_{mk} = A_k - A_{dk}, B_{mk} = B_k - B_{dk}$.

Assume that matrix $C = diag\{C_j\}, j = \{1, 2, ..., M\}$ is block diagonal with dimensions corresponding to subsystems of large-scale system. We assume that each subsystem can be stabilized by output feedback PI controller in the following form

$$u_j(t) = K_{Pj} y_j(t) + k_{Ij} \int y_j(t) dt = K_{Pj} C_j x_j(t) + K_{Ij} z_j(t) \tag{4}$$

where $u_j(t) \in R^{m_j}$ is input vector of $j-th$ subsystem, $\sum_{j=1}^{M} m_j = m$; $y_j(t) \in R^{m_j}$ is output vector of $j-th$ subsystem ; $x_j(t) \in R^{n_j}$ is state vector of $j-th$ subsystem, $\sum_{j=1}^{M} n_j = n$; and $z_j(t) \in R^{m_j}, z_j(t) = \int y_j(t) dt$

Consider $x^T_j(t) := \begin{bmatrix} x_j(t) & z_j(t) \end{bmatrix}^T, C_j := diag\{C_j, I_j\}$ and after some small deriving steps, the local algorithm control of form (4) can be rewritten as follows

$$u_j(t) = F_j C_j x_j(t) \tag{5}$$

Matrices $A(\xi), B(\xi)$ of system (1) respecting to extending state $x^T_j(t) := \begin{bmatrix} x_j(t) & z_j(t) \end{bmatrix}^T$ extend to (without change of denotation)

$$A(\xi) := \begin{bmatrix} A(\xi) & 0 \\ C & 0 \end{bmatrix}, \quad B(\xi) := \begin{bmatrix} B(\xi) \\ 0 \end{bmatrix} \tag{6}$$

For the complex system, the control algorithm (5) can be rewritten as follows

$$u(t) = F y(t) = F C x(t) \tag{7}$$

where $F = diag\{F_j\}, C = diag\{C_j\}, j = \{1, 2, ..., M\}$
For the complex closed-loop system are obtained

$$\dot{x}(t) = \left(A_{dc}(\xi) + A_{mc}(\xi) \right) x(t) \tag{8}$$

where

$$\begin{aligned} A_{dc}(\xi) &= \sum_{k=1}^{N} \xi_k diag\{A^k_{cjj}\}_{j=\{1,2,...,M\}}, \\ A^k_{cjj} &= A^k_{jj} + B^k_{jj} F_j C_j \end{aligned} \tag{9}$$

$$\begin{aligned} A_{mc}(\xi) &= \sum_{k=1}^{N} \xi_k \{A^k_{cij}\}_{i,j=\{1,2,...,M\}}, \\ A^k_{cij} &= \begin{cases} 0, & i = j \\ A^k_{ij} + B^k_{ij} F_j C_j, & i = \{1, 2, ..., M\}, i \neq j \end{cases} \\ j &= \{1, 2, ..., M\} \end{aligned} \tag{10}$$

Simultaneously, with the system (8) we consider the following auxiliary complex system

$$\dot{x}(t) = \left(G_d(\xi) + G_m(\xi) \right) x(t) \tag{11}$$

where

$$\begin{aligned} G_d(\xi) &= \sum_{k=1}^{N} \xi_k diag\{-\gamma^k_{jj}\}, \gamma^k_{jj} > 0 \\ G_m(\xi) &= \sum_{k=1}^{N} \xi_k \{\rho^k_{ij}\}_{i,j=\{1,2,...,M\}} \\ \rho^k_{ij} &= \begin{cases} 0 & i = j \\ \rho^k_{ij} > 0 & i = \{1, 2, ..., M\}, i \neq j \end{cases} \\ j &= \{1, 2, ..., M\} \end{aligned}$$

We introduce the following results [Lancaster and Tismenetsky (1985), Fiedler (1981)]:

Lemma 1 *(Geršgorin Circle) Let* $A = \{a_{ij}\}_{i,j=\{1,...,n\}}$ *and assume that* $a_{ii} > 0$ *for each i and* $a_{ij} \leq 0, i \neq j$. *If A is positive diagonal dominant, that is* $a_{ii} > \sum_{j=1,j\neq i}^{n} |a_{ij}|$ *then A is a M-matrix. Note that, if matrix A is negative diagonal dominant, then matrix A is stable.*

Lemma 2 *(Generalized Geršgorin Circle) Let* $A = \{A_{ij}\}_{i,j=\{1,...,M\}}$ *if* $c_1, ..., c_M$ *are positive numbers such that the following matrix*

$$\begin{bmatrix} c_1 & - \| A_{12} \| & \dots & - \| A_{1M} \| \\ - \| A_{21} \| & c_2 & \dots & - \| A_{2M} \| \\ \vdots & \vdots & \vdots & \vdots \\ - \| A_{M1} \| & - \| A_{M2} \| & \dots & c_M \end{bmatrix}$$

is M - matrix, then all eigenvalues of matrix A lie in the region

$$D_i = \{z : r(A_{ii} - zI_i) \leq c_i\}, i = \{1, 2, ..., M\}$$

where $r(.)$ *is regularity of square matrix.* $r(A) = 0$ *if A is singular and* $r(A) = \left(\| A^{-1} \| \right)^{-1}$ *if A is nonsingular matrix.*

From the observation of above Lemmas, we can conclude

1. Due to Geršgorin Circle the stability of system in k-vertex is guaranteed if

$$\gamma_{jj}^k \geq \sum_{i=1;i\neq j}^{M} \rho_{ij}^k, j = \{1, 2, ..., M\}, k = \{1, 2, ..., M\}. \quad (12)$$

2. **a)** Concerning system (8) and let $G = A_{dc}(\xi)$ and $H = A_{mc}(\xi)$. If system G is robustly stable then $G + H$ will robustly stable if $0 < r < abs\left(max\left(real\left(eig(G) \right) \right) \right)$. To reach such value of r one can try to minimize the $\| H \|, cond(G) \to 1$ and $max\left(\left(real\left(eig(G) \right) \right) \right) < 0$

   b) Concerning system (11) and let $G = G_d(\xi)$ and $H = G_m(\xi)$. If system G is robustly stable then $G + H$ will be robustly stable if $0 < r < abs\left(max\left(real\left(eig(G) \right) \right) \right)$. To obtain above results, due to Geršgorin $\rho_{ij}^k \to min; i, j = \{1, 2, ..., M\}; i \neq j; k = \{1, 2, ..., N\}$ and $\gamma_{jj}^k \to max, j = \{1, 2, ..., M\}; k = \{1, 2, ..., N\}$

Above Lemmas and observation give up the result assertion in the section 3.

Main Result

Assertion 1 *Let* γ_{jj}^k *to be stability degree of j-subsystem for k-vertex and* $\rho_{ij}^k = \| A_{ij}^k + B_{ij}^k F_j C_j \|$*;i,j={1,2,...,M}, if for the system (8), the condition (12) holds,the system is stable in* $k-vertex$, $k = \{1, 2, ..., N\}$.

From above, the following steps may give positive results to robustly stabilize the closed-loop large-scale system.

1. *Design the robust controller with gain matrix* F_j *for j-subsystem such a way that (12) holds and the following subsystem is stable*

$$A_{cjj}^k + \gamma_{jj}^k I, j = \{1, 2, ..., M\}, k = \{1, 2, ..., N\}$$

2. *Design a gain matrix* F_j *so that the following condition holds*

$$\begin{bmatrix} (\rho_{ij}^k)^2 I & (A_{cij}^k)^T \\ A_{cij}^k & I_{ij} \end{bmatrix} \geq 0; i, j = \{1, 2, ..., M\}; i \neq j; k = \{1, 2, ..., N\}$$

3. *Design a gain matrix* F_j *so that* $trace(D_j^k)$ *is minimized and where* $D_j^k = diag\{\rho_{ij}^k\}, i \neq j$

4. *When all subsystems are robust stable checked the robust stability of complex system.*

Unfortunately, we have to conclude that above procedure do not guarantee the stability of complex system, but above procedure gives the way how we can obtain the robust stability of complex system.

Example

In this example we consider the linear model of four cooperating DC motors. The problem is to design four local PI controllers for large scale DC system which will guarantee robust stability and performance of the closed-loop uncertain system. The system model is given by (1) and (2) with a time invariant matrix of 16 order type affine uncertain structure with 4 input and 4 output variables. The goal of design procedure is to design 4 PI controllers which guarantee the robustness properties and performance for closed-loop system.

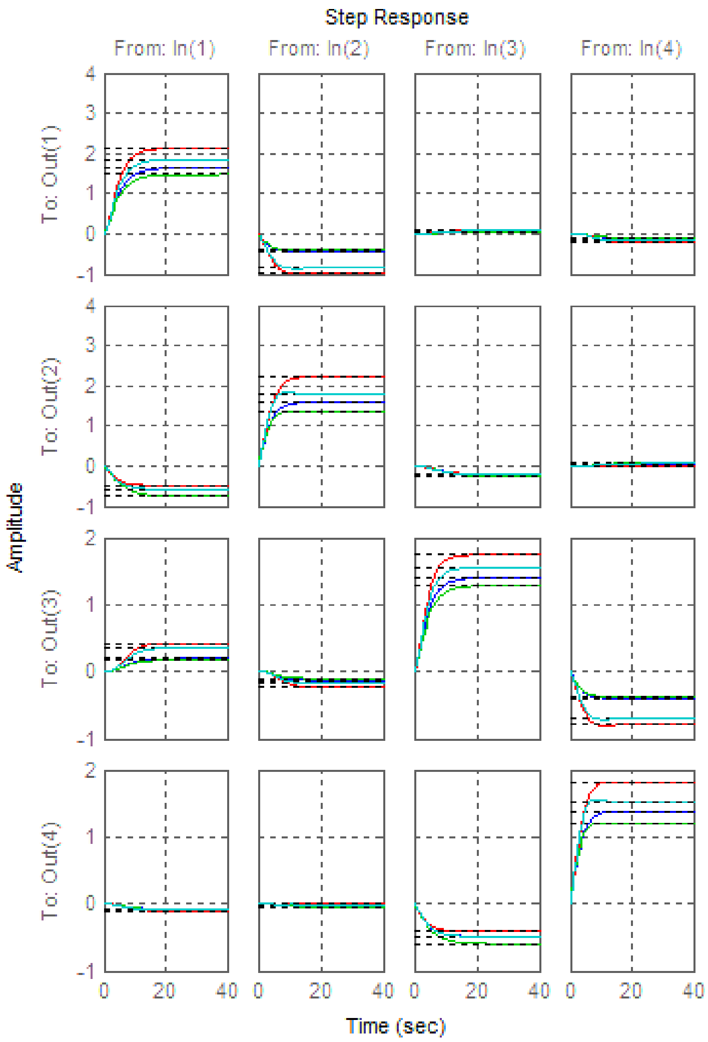

Fig. 1: Step response of system.

Decentralized PI controller has the following parameters

$$K_P = diag\{-0.5619, -0.2429, -0.7033, -0.3685\}$$
$$K_I = diag\{-0.1929, -0.1665, -0.2758, -0.2336\}$$

For the first step,V-K iteration procedure in LMI shows $t_{min} = -0.06122 < 0$, it means that closed-loop system with above PI controller is robust stable with performance.

The result is verified by simulation of closed-loop feedback system obtained in figure (2).

Conclusion

In this paper, a new approach to design robust output feedback PI controller for complex large-scale systems with state output decentralized structure. The proposed design method is based on the Generalized Geršgorin Theorem and the V-K iteration method to design robust PI controller guaranteeing feasible performance achieved in subsystems for the full system and therefore the proposed method excludes limit of system order in LMI/BMI solution.

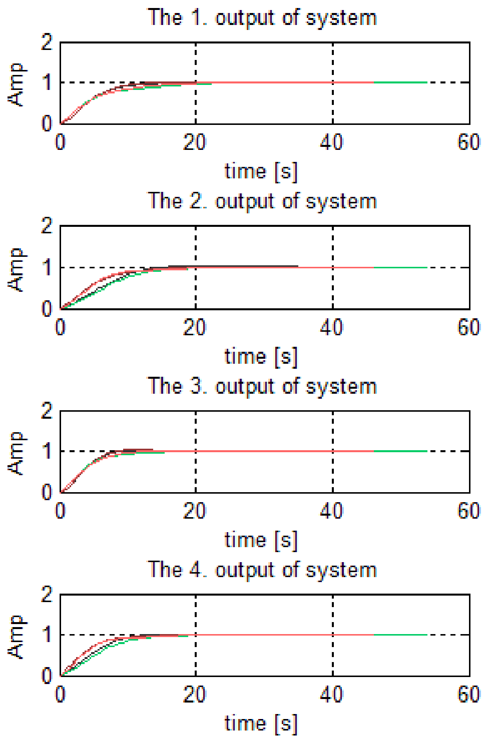

Fig. 2: Output signals of closed-loop feedback system.

Robust decentralized PI controller has been designed using the polytopic description of the uncertain system and applying the robust optimal control design procedure with extended cost function to state-space subsystems generated in each vertex of the polytopic uncertainty domain.

The main advantage of the proposed approach is that the order of the PI design procedure reduces to the order of the particular subsystem.

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