Analytical derivation of friction parameters for FEM calculation of the state of stress in foundation structures on undermined territories

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When calculating the state of stress in a structure caused by relative strain of landscape which is a result of undermining, the structure is often deformed in order to create the specific situation. Each part of the structure resists the strain in a difference way. This depends on places where the structure is in contact with soil environment. When calculating the 3D foundation structures by means of the Finite Element Method (FEM), it is necessary to determine the soil environment resistance.

For that purpose, most FEM software applications enable now to enter the friction parameters C1x and C1y. Unlike C1z which resists the structure in the direction perpendicular to the element's plane, these parameters are applied in the central line plane of a slab and rod element.

***Key words**: friction parameters, FEM calculation, foundation structures*

Introduction

When calculating the 3D foundation structures by means of the Finite Element Method (FEM), it is necessary to determine the soil environment resistance [11]. For that purpose, most FEM software applications enable now to enter the friction parameters C_{1x} and C_{1y}. Unlike C_{1z} which resists the structure in the direction perpendicular to the element's plane, these parameters are applied in the central line plane of a slab and rod element, for instance [6], [7], [10], [13] a [21].

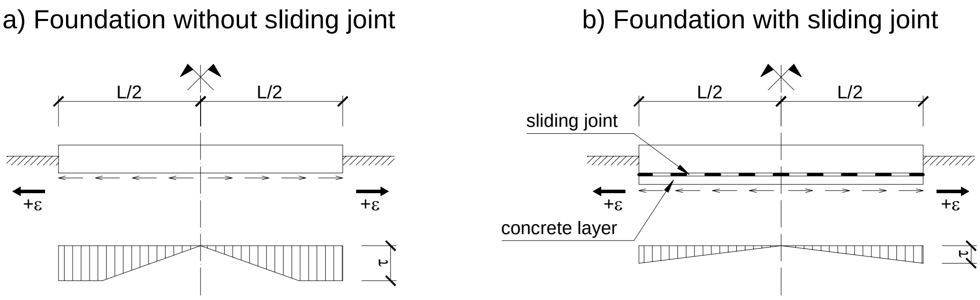

Fig.1. Foundation Structure on the Subsoil Exposed to Strain:
a) without a slide joint, b) with a slide joint

In practical designing works an issue is reliable determination of those parameters. This paper solves the task analytically and provides an numerical example. Numerical approach of solution is evident from [5], [12].

Analytical solution

Differential conditions of the balance are given by the balance conditions for the acting forces in the horizontal direction.

$$\sum F_{ix} = 0 \tag{1}$$

that are determined for a differential element, see Fig. 1.

$$-N_x + N_x + dN_x + p_x.dx - C_{1x}.u.dx = 0 \tag{2}$$

where the friction forces t_x are positively correlated with the friction parameters C_{1x} and shift parameter u

$$t_x = C_{1x}.u \tag{3}$$

Having modified the equation (2), one obtains

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$$\frac{dN_x}{dx} = -p_x + C_{1x}.u \tag{4}$$

Another equation is obtained from a 1D physical equation – Hooke's law with the following relation for the normal force N_x and axial deformation of a rod

$$\sigma_c = E_c.\varepsilon_c \tag{5}$$

Having substituted the stress σ_c and relative strain ε_b (6)

$$\sigma_c = \frac{N_x}{A_c} \quad \varepsilon_c = \frac{du}{dx} \tag{6}$$

in (5) and having derivating the both sides, one obtains

$$\frac{dN_x}{dx} = E_c.A_c.\frac{du^2}{dx^2} \tag{7}$$

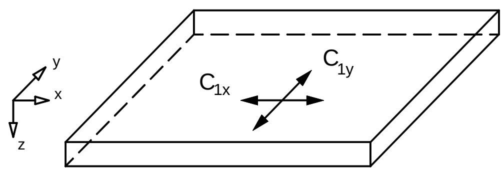

Fig. 2. Orientation of the Friction Parameters C_{1x}, C_{1y}.

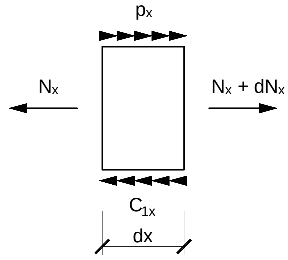

Fig. 3. Differential Element of Balance.

Comparison of the both derivations of the normal force (4) and (7) results in a basic differential equation of a rod which is exposed by an axial force and where friction of the environment is taken into account.

$$E_c.A_c.\frac{du^2}{dx^2} - C_{1x}.u = -p_x \tag{8}$$

It is advisable for the analytical solution to divide (8) with the rigidity $E_c.A_c$ and to introduce substitution there

$$\alpha^2 = \frac{C_{1x}}{E_c.A_c} \tag{9}$$

The final differential equation which is suitable for the analytical solution is then

$$\frac{du^2}{dx^2} - \alpha^2.u = -\frac{p_x}{E_c.A_c} \tag{10}$$

In the first step, it is necessary to solve the homogeneous equation (10) where the right side is zero, this means

$$\frac{du^2}{dx^2} - \alpha^2.u = 0 \tag{11}$$

The solution should be as follows

$$u = e^{r.x} \tag{12}$$

and the second derivation of the function is

$$\frac{du^2}{dx^2} = r^2.e^{r.x} \tag{13}$$

Substitution of the function (12) and derivation of the function (13) in a differential equation (11) and simplification of $e^{r.x}$ results in the following characteristic equation

$$r^2 - \alpha^2 = 0 \tag{14}$$

Having solving the equation, two roots are obtained

$$r_{1,2} = \pm\alpha \tag{15}$$

The final solution to the longitudinal strain in

$$u(x) = A_1.e^{\alpha.x} + A_2.e^{-\alpha.x} \tag{16}$$

where boundary conditions of the task should be used in order to solve the constants A_1, A_2 which are unknown so far.

If the right side of the equation is not zero (this means, if the rod is axially loaded $p_x \neq 0$), a constant variation method, for instance, should be used in order to solve the non-homogeneous equation.

Let us also assumed that $p_x = 0$ applies along the rod. This results in the solution to (11).

Boundary conditions

The following two boundary conditions, see Fig. 2, result from the nature of the task:

- for x=0 strain should be u(0) = 0,
- for x=L the relative strain (this means, derivation of u(L)) is known.

The first boundary condition can be substituted directly in (16), while the other boundary condition results from the modified Hooke's law (5) a (6) which includes now effects of relative strain ε_{max}

$$N_x = \left(\frac{du}{dx} - \varepsilon_{\max}\right).E_c.A_c \tag{17}$$

Having modified (17) and assuming $F_x = N_x$ (the specified axial force at the end of the rod), the second condition is:

$$\frac{du}{dx} = \frac{F_x}{E_c.A_c} + \varepsilon_{\max} \tag{18}$$

In order to express (18), it is necessary to derive the solution to the horizontal strain in the rod axis from (16)

$$\frac{du(x)}{dx} = A_1.\alpha.e^{\alpha,x} - A_2.\alpha.e^{-\alpha,x} \tag{19}$$

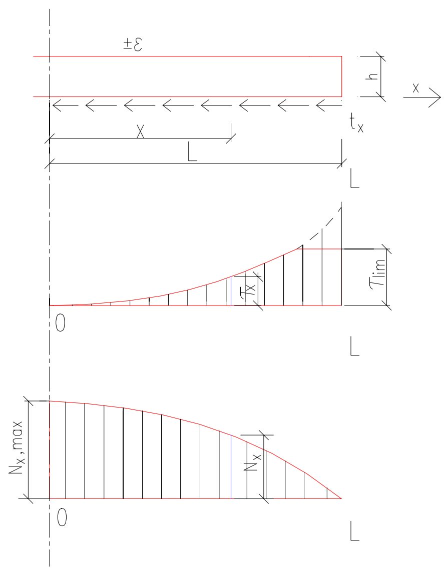

Fig. 4. Distribution of shearing stress and normal forces.

The substitution of the both boundary conditions in (16) and derivation (19) gives a system of 2 linear equations for 2 unknown quantities A_1, A_2:

$$0 = A_1.e^{\alpha,0} + A_2.e^{-\alpha.0} \tag{20}$$

$$\frac{F_x}{E_c.A_c} + \varepsilon = A_1.\alpha.e^{\alpha.L} - A_2.\alpha..e^{-\alpha.L} \tag{21}$$

Having solved (20) and (21) one obtains the following relation for the unknown constants:

$$A_{1,2} = \pm \frac{\frac{F_x}{E_c.A_c} + \varepsilon_{\max}}{\alpha.e^{\alpha,L} + \alpha.e^{-\alpha.L}} \tag{22}$$

The final relation for strain in a rod which is loaded with the axial force F_x in x = L and strain ε_{max} is obtained after introducing the constants (22) into the solution to (16) and after modification of

$$u(x) = \frac{\frac{F_x}{E_c.A_c} + \varepsilon_{\max}}{\alpha.e^{\alpha.L} + \alpha.e^{-\alpha.L}}.\left(.e^{\alpha.x} + e^{-\alpha.x}\right) \tag{23}$$

Derivation of (23) gives

$$\frac{du(x)}{dx} = \frac{\frac{F_x}{E_c.A_c} + \varepsilon_{\max}}{\alpha.e^{\alpha.l} + \alpha.e^{-\alpha.l}}.\left(\alpha.e^{\alpha.x} + \alpha.e^{-\alpha.x}\right) \tag{24}$$

The normal forces can be determined by introducing (24) into (17)

$$N_x = \frac{F_x + E_c.A_c.\varepsilon_{\max}}{\alpha.e^{\alpha.l} + \alpha.e^{-\alpha.l}}.\left(\alpha.e^{\alpha.x} + \alpha.e^{-\alpha.x}\right) - E_c.A_c.\varepsilon_{\max} \tag{25}$$

If the maximum axial force in the middle of the foundation structure is know (this can be solved, for instance, in line with ČSN 73 0039 [1]), the input parameters E_c, A_c, L and ε_{max} can be used in (25) for determination of the friction parameter C_{1x} for x = 0

$$\begin{aligned} N_x &= \frac{F_x + E_c.A_c.\varepsilon_{\max}}{\alpha.e^{\alpha.l} + \alpha.e^{-\alpha.l}}.\left(\alpha.e^{\alpha..0} + \alpha.e^{-\alpha.0}\right) - E_c.A_c.\varepsilon_{\max} = \\ &= \frac{F_x + E_c.A_c.\varepsilon_{\max}}{\alpha.e^{\alpha.l} + \alpha.e^{-\alpha.l}}.2.\alpha - E_c.A_c.\varepsilon_{\max} = \\ &= \frac{F_x + E_c.A_c.\varepsilon_{\max}}{e^{\alpha.l} + e^{-\alpha.l}}.2 - E_c.A_c.\varepsilon_{\max} \end{aligned} \tag{26}$$

When dealing with (26) with the unknown parameter α it is advisable to modify the equation as follows

$$N_{x,\max} + E_c.A_c.\varepsilon_{\max} = \frac{F_x + E_c.A_c.\varepsilon_{\max}}{\alpha.e^{\alpha.l} + \alpha.e^{-\alpha.l}}.2 \tag{27}$$

and then as follows

$$e^{\alpha.L} + e^{-\alpha.L} = \frac{F_x + E_c.A_c.\varepsilon_{\max}}{N_{x,\max} + E_c.A_c.\varepsilon_{\max}}.2 \tag{28}$$

Let us assume the following hyperbolic function cosh(x)

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \tag{29}$$

(28) can be modified then as follows

$$\cosh(\alpha.L) = \frac{F_x + E_c.A_c.\varepsilon_{\max}}{N_{x,\max} + E_c.A_c.\varepsilon_{\max}} \tag{30}$$

The inversion function of cosh(x) is an arc-hyperbolic function argcosh(x), this means the argument of the hyperbolic cos x. Having modified (30) again, one obtains

$$\alpha = \frac{1}{L}.\arg\cosh\left(\frac{F_x + E_c.A_c.\varepsilon_{max}}{N_{x,max} + E_c.A_c.\varepsilon_{max}}\right) \tag{31}$$

The friction parameter C_{1x} is obtained by substitution (9)

$$C_{1x} = E_c.A_c.\alpha^2 \tag{32}$$

which, after introduction of α from (31) gives

$$C_{1x} = E_c.A_c.\left[\frac{1}{L}.\arg\cosh\left(\frac{F_x + E_c.A_c.\varepsilon_{max}}{N_{x,max} + E_c.A_c.\varepsilon_{max}}\right)\right]^2 \tag{33}$$

(33) can be modified if the arc-hyperbolic function argcosh(x) is described by means of a logarithm.

For x > 1

$$\arg\cosh(x) = \ln\left(x + \sqrt{x^2 - 1}\right) \tag{34}$$

The relation for the friction parameter C_{1x} (33) can be modified as follows

$$C_{1x} = E_c.A_c.\left[\frac{1}{L}.\ln\left(\frac{F_x + E_c.A_c.\varepsilon_{max}}{N_{x,max} + E_c.A_c.\varepsilon_{max}} + \sqrt{\left(\frac{F_x + E_c.A_c.\varepsilon_{max}}{N_{x,max} + E_c.A_c.\varepsilon_{max}}\right)^2 - 1}\right)\right]^2 \tag{35}$$

or as follows

$$C_{1x} = E_c.A_c.\left[\frac{1}{L}.\ln\left(B_x + \sqrt{B_x^{\ 2} - 1}\right)\right]^2 \tag{36}$$

where the constant is

$$B_x = \frac{F_x + E_c.A_c.\varepsilon_{max}}{N_{x,max} + E_c.A_c.\varepsilon_{max}} \tag{37}$$

Example – calculation of a friction parameter

In order to validate the proposed solution, calculations were carried out for a reinforced concrete foundation slab described in [4], see Fig. 5. The solution was performed for the constant friction parameter C_{1x} by means of (33) and for non-linear C_{1x} by means of FEM, see [12].

The relative strain is $\varepsilon_{max} = 5{,}0.10^{-3}$ and correction coefficient is $\mu_\varepsilon = 0.85$ for the applicable mining environment. The subsoil is compacted sand with the internal friction angle $\varphi_{ef} = 32^o$, cohesion $c_{ef} = 0$, modulus of elasticity $E_{def} = 20$ MPa and Poisson ratio $\nu = 0.3$.

The designed average contact stress in the foundation joint is $\sigma_{vd} = 240$ kPa.

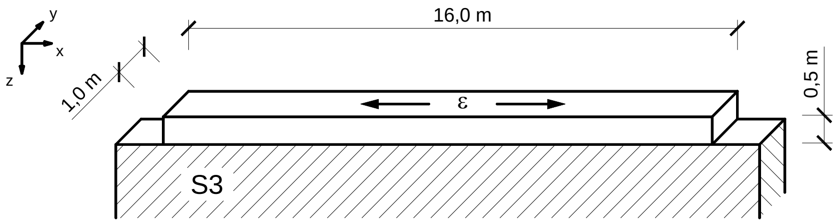

Fig. 5. Specification for the Comparative Example.

For the slab length L = 16.0 m, slab width b = 1.0 m and the relative strain ε_{max} the depth of the foundation soil which is still influenced by the foundation structure is:

$$a = 0{,}75.L^{0{,}56}.\left(1 - e^{-0{,}94.b^{0{,}53}}\right) = 0{,}75.16{,}0^{0{,}56}.\left(1 - e^{-0{,}94.1{,}0^{0{,}53}}\right) = 2{,}159 \quad m \tag{43}$$

The solution pursuant to ČSN 73 0039 and comments [1] results in the following maximum shearing stress at the end of the beam [4]

$$\tau_{xz,max} = 75.0\ kPa$$

and in the following maximum tensile stress inside the slab

$$N_{x,max} = 237.8\ kN$$

When calculating the foundation structure which is loaded by relative strain ε_{max} in the subsoil with the friction parameter C_{1x}, one should obtain at least the same maximum force $N_{x,max}$. This should be in line with the calculated friction parameters C_{1x} which is determined from the derived (33) where following values are introduced:

$F_x = 0,\ L = 8.0\ m,\ E_c = 27.10^6\ kPa,\ A_c = 0.5\ m^2, \varepsilon_{max} = 5.10^{-3}\ a\ N_{x,max} = 237.8\ kN$

Then, the constant friction parameter C_{1x} is:

$$C_{1x} = E_c.A_c.\left[\frac{1}{L}.\operatorname{arccos}h\left(\frac{F_x + E_c.A_c.\varepsilon_{max}}{N_{x,max} + E_c.A_c.\varepsilon_{max}}\right)\right]^2 =$$

$$= 27.10^6.0{,}5.\left[\frac{1}{8{,}0}.\operatorname{arccos}h\left(\frac{0 + 27{,}0.10^6.0{,}5.5.10^{-3}}{237{,}8 + 27{,}0.10^6.0{,}5.5.10^{-3}}\right)\right]^2 = 1490{,}6 \quad kN.m^{-2} \tag{44}$$

When the average friction parameters is assumed to be the ratio (the maximum shearing stress $\tau_{xz,max}$ = 75.0 kPa to the maximum strain under the edge of the foundation)

$$u_{max} = \varepsilon_{max}.L = 5.10^{-3}.8{,}0 = 0{,}040 m$$

the friction parameter is considerably higher

$$C_{1x} = \frac{\tau_{xz,max}}{u_{max}} = \frac{75{,}0}{0{,}040} = 1875{,}0 \quad kN.m^{-2}$$

Fig. 6 show results for the constant and non-linear friction parameters C_{1x} [12] along the slab length.

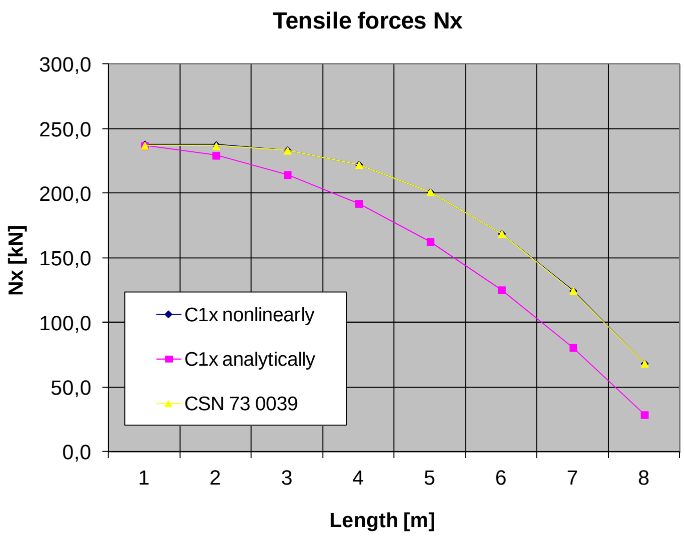

Fig. 6. Tensile forces along the rod.

Conclusion

This paper discusses procedures and relations needed for calculation of the friction parameters C_{1x} which should be entered as input parameters for foundation structures located in subsoil where undermining strain occurs [1], [25], [26]. The proposed procedures can be applied to other types of the deformation load, for instance because of the concrete shrinkage or temperature changes [8]. In order to find out the maximum forces it is enough to know the constant course of C_{1x} for the entire foundation structure. If a more precise course of deformation and axial forces is needed, it is essential to determine the non-linear course of the friction parameter C_{1x} in the individual members [12]. In case of 2D structures, the calculation takes into account resistance of environment in the second direction which is characterised by the friction parameter C_{1y} [11]. The solution to such 2D structure shows the progress of strain and internal forces even those parts of the structure which are above soil and does not touch soil at all. This solution can be used also for rheological sliding joints which decrease considerably friction resistance between the foundations and subsoil [3], [8], [9], [20] or for prestressed foundation or floor structures [24].

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References

[1] CSN 73 0039 Design constructed facilities in mining subsidence areas. *Basic requirements. 10/1989, a-06/1991.*

[2] Asadi, A., Shakhriar, K., Goshtasbi, K.: Profiling function for surface subsidence prediction in mining inclined coal seams (2004), *Journal of Mining Science, 40 (2), pp. 142-146, doi: 10.1023/B:JOMI.0000047856.91826.76*

[3] Balcárek, V., Bradáč, J.: Using asphalt sealing felts for sliding joints in structures on undermined territories, *Pozemní stavby 2 – 1982.*

[4] Bradáč, J.: Effects of undermining and protection of constructions, *Expert – technické nakladatelství Ostrava, 1996.*

[5] Cajka, R.: Determining the friction parameter Clx between the foundation and subsoil under strain (2000), *Mezinárodní konference Hornická Příbram ve vědě a technice, Příbram*.

[6] Cajka, R.: Soil – structure interaction in case of exceptional mining and flood actions, *COST 12 – Final Conference Proceedings, 20th - 22nd January 2005, University of Innsbruck, Austria, ISBN 04 1536 609 7*.

[7] Cajka, R., Manasek, P.: Building Structures in Danger of Flooding. *IABSE Conference New Delhi, India 2005: Role of Structural Engineers towards Reduction of Poverty. New Delhi, India, pp. 551-558, ISBN 978-3-85748-111-6, WOS: 000245746100072*.

[8] Cajka, R., Manasek, P.: Finite Element Analysis of a Structure with a Sliding Joint Affected by Deformation Loading, *In B.H.V. Topping, (Editor), "Proceedings of the Eleventh International Conference on Civil, Structural and Environmental Engineering Computing", Civil-Comp Press, Stirlingshire, UK, Paper 18, 2007. ISBN 978-1-905088-17-1, doi:10.4203/ccp.86.18.*

[9] Cajka, R., Manasek, P.: Numerical analysis of the foundation structures with sliding joint. *Eleventh East Asia-Pacific Conference on Structural Engineering&Construction - Building a Sustainable Environment, Taipei, Taiwan, 19. – 21.11. 2008. pp. 716-717. Proceedings and CD, ISBN 978-986-80222-4-9*

[10] Cajka, R.: Accuracy of Stress Analysis Using Numerical Integration of Elastic Half-Space (2013), *Applied Mechanics and Materials, 300-301, pp. 1127-1135. Trans Tech Publications, Switzerland, ISSN: 16609336, ISBN: 978-303785651-2, doi: 10.4028/www.scientific.net/AMM.300-301.1127*.

[11] Cajka, R., Labudek, P., Burkovic, K., Cajka, M.: Golf Club Structure and Foundation with Slide Joint on the Undermined Territory. *6th WSEAS International Conference on Engineering Mechanics, Structures, Engineering Geology (EMESEG '13), WSEAS / NAUN International Conferences, Cambridge, UK, February 20-22, 2013, ISSN 2227-4588, ISBN 978-1-61804-165-4*.

[12] Cajka, R.: Horizontal Friction Parameters in Soil – Structure Interaction Tasks. *Advanced Materials Research, Vol. 818 (2013), pp 197-205, Trans Tech Publications, Switzerland, doi:10.4028/www.scientific.net/AMR.818.197.*

[13] Cajka, R.: Analysis of Stress in Half-space using Jacobian of Transformation and Gauss Numerical Integration. *Advanced Materials Research, Vol. 818 (2013), pp 178-186, Trans Tech Publications, Switzerland, doi:10.4028/www.scientific.net/AMR.818.178*.

[14] Can, E., Kuşcu, Ş., Kartal, M.E.: Effects of mining subsidence on masonry buildings in Zonguldak hard coal region in Turkey (2012), *Environmental Earth Sciences, 66 (8), pp. 2503-2518. ISSN: 18666280, doi: 10.1007/s12665-011-1473-2*

[15] Cui, X.M., Li, C.Y., Hu, Q.F., Miao, X.X.: Prediction of surface subsidence due to underground mining based on the zenith angle (2013). *International Journal of Rock Mechanics and Mining Sciences, 60, pp. 246-252, ISSN: 13651609, doi: 10.1016/j.ijrmms.2012.12.036*

[16] Cui, X., Miao, X., Wang, J., Yang, S., Liu, H., Song, Y., Liu, H., Hu, X.: Improved prediction of differential subsidence caused by underground mining (2000), *International Journal of Rock Mechanics and Mining Sciences, 37 (4), pp. 615-627. ISSN: 13651609, doi: 10.1016/S1365-1609(99)00125-2*

[17] Huayang, D., Jinzhuang, W., Meifeng, C., Lixin, W., Zengzhang, G.: Seam dip angle based mining subsidence model and its application (2002), *International Journal of Rock Mechanics and Mining Sciences, 39 (1), pp. 115-123. doi: 10.1016/S1365-1609(02)00008-4*

[18] Chai, H.: Study on the deformation and stability of building foundations in mining subsidence areas (2012), *Applied Mechanics and Materials, 166-169, pp. 1967-1970. ISSN: 16609336, ISBN: 978-303785421-1, doi: 10.4028/www.scientific.net/AMM.166-169.1967*

[19] Cheng, G.M., Liu, T.Z., Zhi, B.: Numerical modeling of subsidence-induced stress on the pipeline in steep seam mining (2014). *Applied Mechanics and Materials, 448-453, pp. 3863-3868, ISSN: 16609336 ISBN: 978-303785912-4, doi: 10.4028/www.scientific.net/AMM.448-453.3863*

[20] Janulikova, M., Stara, M.: Reducing the Shear Stress in the Footing Bottom of Concrete and Masonry Structures, *Procedia Engineering, Volume 65, 2013, Pages 284-289, ISSN 1877-7058, doi: 10.1016/j.proeng.2013.09.044*.

[21] Janda, T., Sejnoha, M., Sejnoha, J.: Modeling of soil structure interaction during tunnel excavation: An engineering approach. *Advances in Engineering Software, 62-63, pp. 51-60., doi: 10.1016/j.advengsoft.2013.04.011, 2013*.

[22] Kalab, Z. Lednicka, M.: Foundation conditions of buildings in undermined areas: An example of evaluation. *(2012). Acta Geophysica, 60 (2), pp. 399-409. ISSN: 18956572, doi: 10.2478/s11600-011-0071-8*

[23] López Gayarre, F., Álvarez-Fernández, M.I., González-Nicieza, C., Álvarez-Vigil, A.E., Herrera García, G.: Forensic analysis of buildings affected by mining subsidence (2010), *Engineering Failure Analysis, 17 (1), pp. 270-285. ISSN: 13506307, doi: 10.1016/j.engfailanal.2009.06.008*

[24] Mynarcik P.: Technology and Trends of Concrete Industrial Floors, *Procedia Engineering, Volume 65, 2013, Pages 107-112, ISSN 1877-7058, doi: 10.1016/j.proeng.2013.09.019*.

[25] Marschalko, M., Treslin, L.: Impact of underground mining to slope deformation genesis at Doubrava Ujala (2009), *Acta Montanistica Slovaca, 14 (3), pp. 232-240*.

[26] Marschalko, M., Fuka, M., Třeslín, L.: Influence of Mining Activity on Selected Landslide in the Ostrava-Karviná Coalfield (2008), *Acta Montanistica Slovaca, 13, (1), pp. 58-65, 2008*.

[27] Qi, X.D., Cheng, X.Z.: Analysis on the surface subsidence regular about paste filling mining under the buildings (2013), *Advanced Materials Research, 734-737, pp. 290-293, ISSN: 10226680 ISBN: 978-303785744-1, doi: 10.4028/www.scientific.net/AMR.734-737.290*

[28] Ren, G., Whittaker, B.N., Reddish, D.J.: Mining subsidence and displacement prediction using influence function methods for steep seams (1989), *Mining Science and Technology, 8 (3), pp. 235-251*.

[29] Saeidi, A., Deck, O., Verdel, T.: Comparison of Building Damage Assessment Methods for Risk Analysis in Mining Subsidence Regions (2013), *Geotechnical and Geological Engineering, 31 (4), pp. 1073-1088. ISSN: 09603182, doi: 10.1007/s10706-013-9633-7*

[30] Swift, G.: Relationship between joint movement and mining subsidence (2013), *Bulletin of Engineering Geology and the Environment, pp. 1-14. ISSN: 14359529 doi: 10.1007/s10064-013-0539-7, Article in Press*.

[31] Unlu, T., Akcin, H., Yilmaz, O.: An integrated approach for the prediction of subsidence for coal mining basins (2013), *Engineering Geology, 166, pp. 186-203, ISSN: 00137952 doi: 10.1016/j.enggeo.2013.07.014*

[32] Woo, K.S., Eberhardt, E., Rabus, B., Stead, D., Vyazmensky, A.: Integration of field characterisation, mine production and InSAR monitoring data to constrain and calibrate 3-D numerical modelling of block caving-induced subsidence (2012), *International Journal of Rock Mechanics and Mining Sciences, 53, pp. 166-178, doi: 10.1016/j.ijrmms.2012.05.008*

[33] Xia, J.W., Yuan, Y.S., Dong, Z.Z.: Mechanics model of ground, independent footing and framework interaction in mining subsidence area (2007), *Zhongguo Kuangye Daxue Xuebao/Journal of China University of Mining and Technology, 36 (1), pp. 33-37. ISSN: 10001964*