

# Application of the MATLAB - Simulink package in the simulation tests on hydrostatic systems

Zenon Jędrzykiewicz<sup>1</sup>, Janusz Pluta<sup>1</sup>, Jerzy Stojek<sup>1</sup>

**Abstract:** The article shows some selected problems related to both modelling and the simulation of hydrostatic systems, by making use of MATLAB-Simulink package. In this purpose there have been considered the basic mathematical models of certain selected elements and phenomena occurring in hydrostatic systems. The models are shown as block diagrams adapted to the package requirements. Afterwards, taking as example a complex hydraulic system - that is a hydrostatic transmission - there has been illustrated the use of the models and elementary diagrams in simulation tests.

**Key words:** hydrostatic transmission, mathematical model, block diagram, simulation tests.

## Introduction

Contemporary design works of hydrostatic systems involves more detailed knowledge of the phenomena that may take places in static and dynamic states of real systems. Here, especially important are simulation tests for being appreciably cheaper and more rapid than experimental ones. The principles of simulation tests are considered for example in Pluta (1987) and Stryczek (1995) etc., however the problems are often simplified and do not take exact mathematical models into consideration like in i.e. (Flaga et al., 1993; Pluta, 1987, 1989, 1993). Simulation tests are particularly useful if model creation of real system is complicated and time-consuming. Hydrostatic systems of mining machines and equipment like heading machine shearer loader or incline winch safety, are good examples here. The determination of static and dynamic characteristics for the above systems is possible and more and more often carried out using the mentioned simulation tests (Pluta, 1989, 1993, 1995; Pluta et al., 1996). General availability to more and more sophisticated computer hardware and better computer software with mathematical - and - simulating programs can enable operational simulations even in very complicated systems. MATLAB-Simulink (MATLAB, 1992; Simulink, 1992) is often used

in simulation tests and that is because of the following advantages:

- an appreciably simplified procedure that exonerates a designer from encoding his mathematical model in the programming language adopted,
- a possibility of a multiple use of the basic mathematical models, gathered at the libraries and combined as needed,
- a possibility of examining systems with models having appreciably extended ranges of coefficients, in particular for hydraulic systems,
- a possibility of comparing theoretical and experimental results.

The article is aimed at presenting some selected examples of modelling and simulation of hydrostatic systems operation by means of MATLAB-Simulink. First, there have been considered the basic mathematical models of the selected elements and phenomena occurring in hydrostatic systems. The models are shown both as block and detailed diagrams adapted to the requirements of the MATLAB-Simulink graphical editor. Then, there has been shown an example illustrating the use of the basic models in the testing of complex systems. The example presents as well a hydrostatic transmission with a variable delivery pump. Such hydrostatic system is used for example in travel drive system of shearer loader (Pluta, 1989, 1996; Podsiadło et al., 1996).

## Mathematical models of a variable delivery positive-displacement pump

<sup>1</sup> Department of Process Control, University of Mining and Metallurgy, Cracow, PL (Received January, 13, 1998)

In the simulation tests we usually make use of the formula for the pump delivery. For a variable delivery pump, an approximate expression (Jędrzykiewicz, 1981; Stryczek, 1995):

$$Q_p = \alpha_p K_{qp} \eta_{vp} \tag{1}$$

where:  $Q_p$  - pump delivery [ $m^3 \cdot s^{-1}$ ],  $\alpha_p$  - displacement angle of either a pump disk or casing [ $^\circ$ ],  $K_{qp}$  - pump delivery coefficient [ $m^3 \text{ per } ^\circ \cdot s^{-1}$ ],  $\eta_{vp}$  - pump volumetric efficiency coefficient [-].

Formula (1) can be illustrated with either a general block diagram as shown in Fig. 1 or a detailed one as in Fig. 2. While making diagrams, it was assumed that  $\alpha_p$  and  $\eta_{vp}$  would be input and  $Q_p$  - an output signal. An analogous procedure was adapted while making the remaining diagrams.

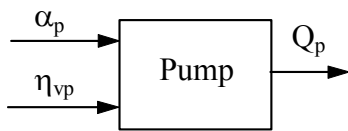


Fig. 1

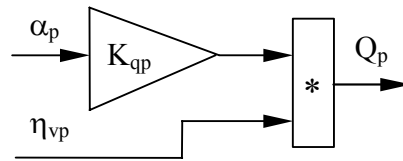


Fig. 2

In the tests is also useful the formula expressing the torque, indispensable for driving a pump. Basing upon (Stryczek, 1995) we get:

$$M_p = \frac{Q_p \Delta p_p}{2\pi n_p \eta_{vp} \eta_{mp}} \tag{2}$$

where:  $\Delta p_p$  - pressure fall in the pump [Pa],  $n_p$  - pump shaft rotational velocity [ $rev \cdot s^{-1}$ ],  $\eta_{mp}$  - coefficient of the pump's mechanico-hydraulic efficiency [-].

Formula (2) can be illustrated either via a general block diagram as shown in Fig. 3 or a detailed one as in Fig. 4.

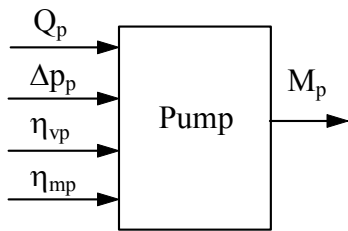


Fig. 3

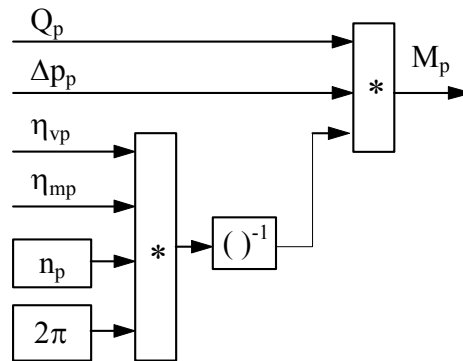


Fig. 4

### Mathematical models for a constant absorptivity displacement engine

A displacement engine is usually described with two formulas. One of it regards absorptivity and the other - its torque. According to (Jędrzykiewicz, 1981; Stryczek, 1995), the engine absorptivity can be put down as follows:

$$Q_h = \frac{K_{gh} \omega_h}{\eta_{vh}} \tag{3}$$

where:  $Q_h$  - engine absorptivity [ $m^3 \cdot s^{-1}$ ],  $K_{qh}$  - engine absorptivity coefficient [ $m^3$ ],  $\omega_h$  - angular velocity of the engine shaft [ $s^{-1}$ ],  $\eta_{vh}$  - coefficient of the engine's volumetric efficiency [-].

By analogy, proceeding as for the pump, we shall arrive at the diagrams shown in Figs 5 and 6. Basing upon (Jędrzykiewicz, 1981; Stryczek, 1995), the torque developed by the engine amounts to:

$$M_h = K_{mh} \Delta p_h \eta_{mh}, \tag{4}$$

where:  $M_h$  - torque [N·m],  $K_{mh}$  - engine's torque coefficient [ $m^3$ ],  $\Delta p_h$  - pressure fall in the engine [Pa],  $\eta_{mh}$  - coefficient of the engine's mechanico-hydraulic efficiency [-]. Respective block diagrams of the engine are shown in Figs 7 and 8.

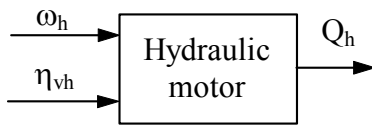


Fig. 5

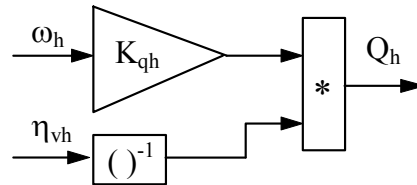


Fig. 6

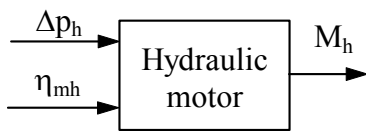


Fig. 7

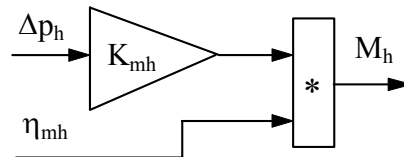


Fig. 8

**A mathematical model for a safety valve**

In simplified considerations one usually analyses the static characteristics of a valve, arising out of its catalogue specifications. In such a case one can make use of two formulas for different operational phases of the valve - when closed and open.

$$Q_z = K_{zb}(p - p_b) \quad \text{for } p > p_b, \tag{5}$$

$$Q_z = 0 \quad \text{for } p \leq p_b, \tag{6}$$

where:  $Q_z$  - flow rate through the valve [ $m^3 \cdot s^{-1}$ ],  $K_{zb}$  - slope coefficient of the valve's static characteristics [ $m^5 \cdot N^{-1} \cdot s^{-1}$ ],  $p$  - system operational pressure [Pa],  $p_b$  - valve opening pressure set while in operation [Pa].

Respective block diagrams are shown in Figs 9 and 10. Sw in Fig. 10 stands for a switch for performing either formula (5) or (6) (Simulink, 1992).

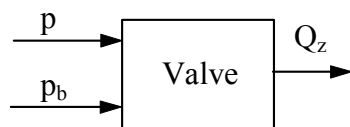


Fig. 9

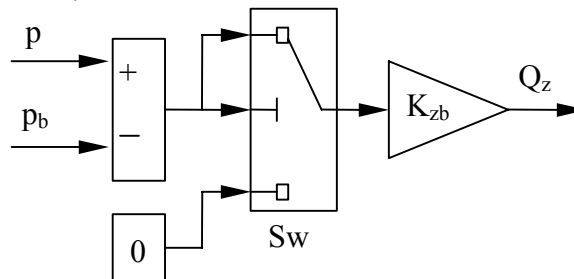


Fig. 10

### A mathematical model for the fluid compressibility effect

The fluid compressibility effect can be put down with an approximate relation (Stryczek, 1995; Jędrzykiewicz, 1981):

$$Q_s = \frac{V_s}{E_s} Dp, \quad (7)$$

where:  $Q_s$  - fluid flow rate related to compressibility [ $\text{m}^3/\text{s}$ ],  $V_s$  - fluid volume subject to pressure effects [ $\text{m}^3$ ],  $E_s$  - fluid bulk modulus [Pa],  $p$  - fluid pressure [Pa],  $D$  - differentiating operator. Relation (7) usually serves for determining pressure values at a known flow rate, therefore:

$$p = \frac{E_s}{V_s} \cdot \frac{1}{D} Q_s, \quad (8)$$

where  $1/D$  - integrating operator. Respective block diagrams are shown in Figs 11 and 12.

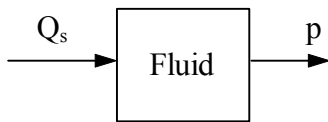


Fig. 11

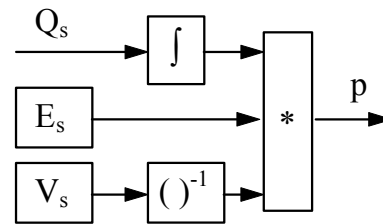


Fig. 12

### A mathematical model for the hydraulic engine load

Torque  $M_h$  generated in an engine is in equilibrium with the moments resulting from engine loading. Generally, it can be put down as follows:

$$M_h = M_I + M_B + M_o \quad (9)$$

or after extending the torques:  $M_I$  - resulting from inertial loads and  $M_B$  - resulting from frictional resistances:

$$M_h = I_h D\omega_h + B_h \omega_h + M_o, \quad (10)$$

where:  $I_h$  - moment of inertia of the engine and machine parts, reduced upon the engine shaft [ $\text{N}\cdot\text{m}\cdot\text{s}^2$ ],  $B_h$  - resistance coefficient of viscous friction in the engine and machine parts, reduced upon the engine shaft [ $\text{N}\cdot\text{m}\cdot\text{s}$ ],  $M_o$  - moment of technological resistance, resulting from the machine operation [ $\text{N}\cdot\text{m}$ ],  $\omega_h$  - angular velocity of the engine shaft [ $\text{s}^{-1}$ ].

Relation (10) is chiefly used for determining the angular velocity of the hydraulic engine shaft, with

$$\omega_h = \frac{1}{D} \left( \frac{M_h - B_h \omega_h - M_o}{I_h} \right) \quad (11)$$

Basing upon (11) one can make block diagrams as in Figs 13 and 14.

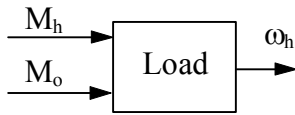


Fig. 13

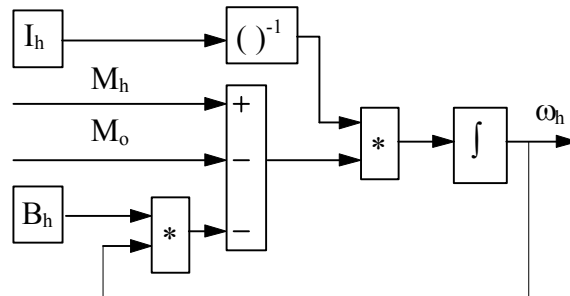


Fig. 14

### A mathematical model for a hydrostatic transmission

A hydrostatic transmission with a variable delivery pump and a constant absorptivity engine has been taken as example for illustrating the models said above. The calculating procedure for the transmission being considered is as shown in Fig. 15.

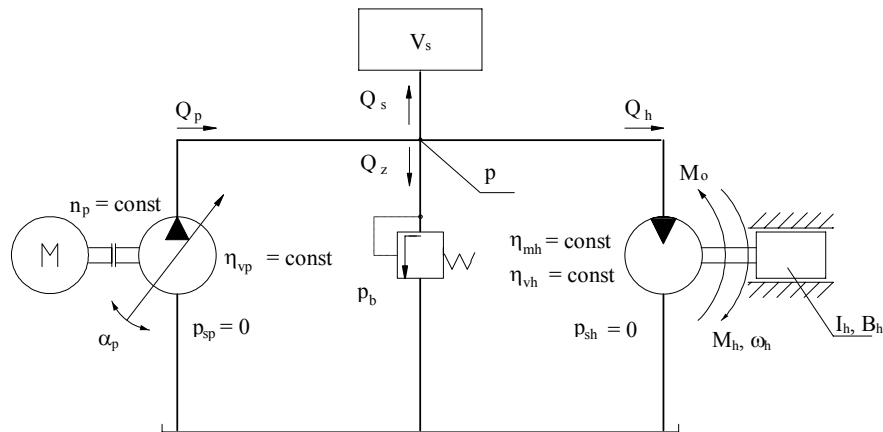


Fig. A5

The mathematical model of the transmission consists of two equations -of the equation for the flow rates' balance and that of the loads' equilibrium. The general form is as follows:

$$Q_p = Q_h + Q_s + Q_z, \quad (12)$$

$$M_h = M_I + M_B + M_o. \quad (13)$$

According to (8) and (11), equations (12) and (13) can be boiled down to the following expression:

$$p = \frac{E_s}{V_s} \cdot \frac{1}{D} (Q_p - Q_h - Q_z), \quad (14)$$

$$\omega_h = \frac{1}{D} \left( \frac{M_h - B\omega_h - M_o}{I} \right) \quad (15)$$

Bearing in mind that  $\Delta p_h = p$  and making use of the graphical editor of the MATLAB-Simulink package (Simulink, 1992), the block diagrams as in Figs 16 and 17, have been obtained. Fig. 16 shows a detailed block diagram regarding the transmission model. This diagram was worked out by referring to the pre-existent principal models of diagrams, collected in the related hydraulic library. Fig. 17 presents a general block diagram, made after having aggregated the detailed one. It illustrates the data loading, input signals and taking out the results.

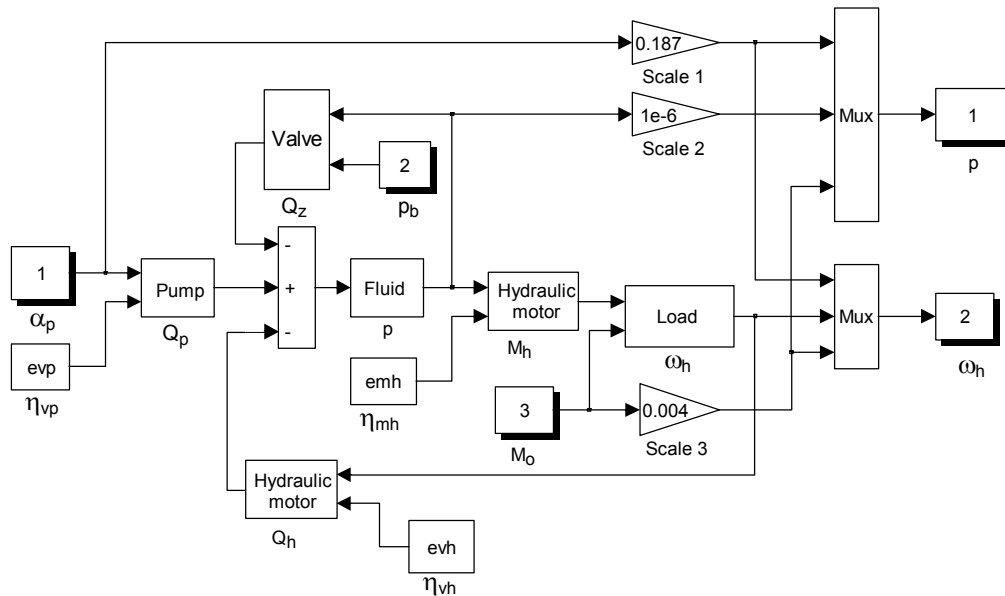


Fig. 16

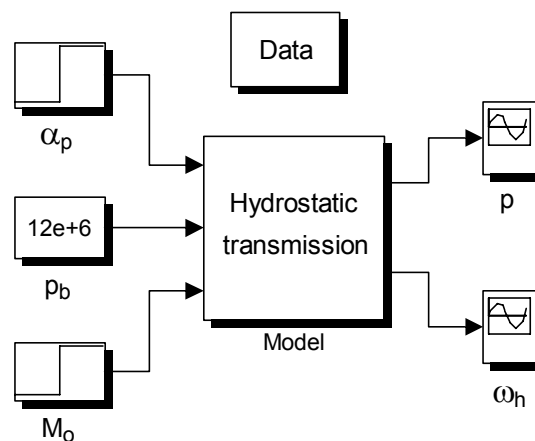


Fig. 17

### Simulation tests

Basing upon either catalogue or experimental data, the following values of the coefficients for the transmission model have been found:  $A_\alpha=16$  [°],  $A_m=250$  [N·m],  $B_n=15$  [N·m·s],  $E_s=1.65e+9$  [Pa],  $I_h=0.04$  [N·m·s<sup>2</sup>],  $K_{qp}=2.6882e-5$  [m<sup>3</sup>per°, s],  $K_{qh}=3.979e-5$  [m<sup>3</sup>],  $K_{mh}=3.979e-5$  [m<sup>3</sup>],  $K_{zb}=0.20e-9$  [m<sup>5</sup>·N<sup>-1</sup>·s<sup>-1</sup>],  $p_b=12.0e+6$  [Pa],  $t_\alpha=0.01$  [s],  $t_m=0.05$  [s],  $V_s=1.4145e-4$  [m<sup>3</sup>],  $\eta_{vp}=0.97$  [-],  $\eta_{vh}=0.93$  [-],  $\eta_{mh}=0.95$  [-]. The tests were carried out for a jump-like change in the pump delivery  $\alpha_p = A_\alpha \cdot 1(t-t_\alpha)$  and a jump-like change in the engine load  $M_o = A_m \cdot 1(t-t_m)$ . The tests results are shown in Figs 18 and 19.

### Summary

Large dynamic changes in the angular velocity and pressure in the transmission pumping duct result from the characteristics obtained. These changes can occur, for example, after a jump-like re-control of the pump delivery and afterwards, following a jump-like increase in the moment of technological resistance. With exception to stationary states, these results are unforeseeable via a conventional static designing and can be a useful indication for a designer of a transmission and other hydrostatic systems. The character of the obtained courses is qualitatively convergent with the research results arrived at in the lab conditions. The results shown were obtained in a relatively easy and rapid way due to the basic advantages of the MATLAB-Simulink package, for example:

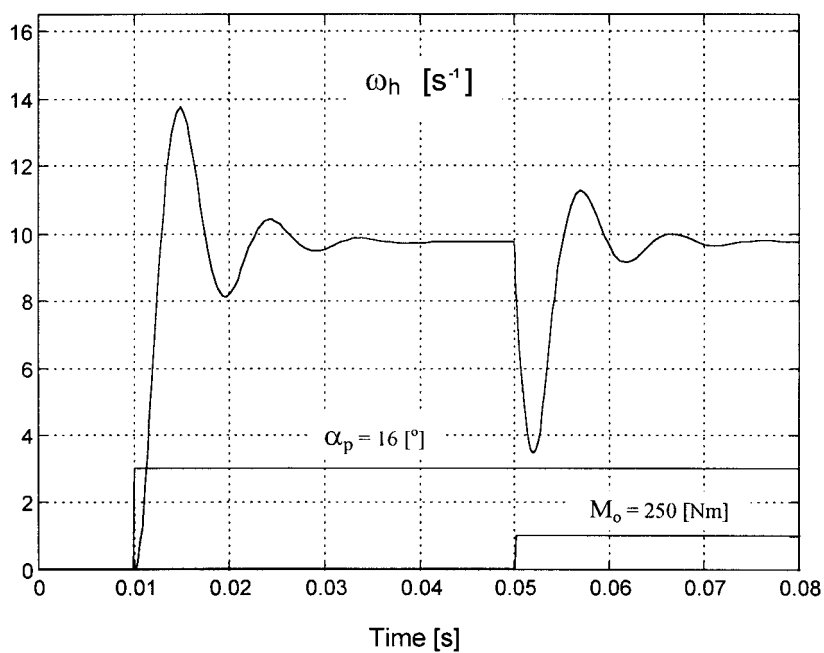


Fig. 18

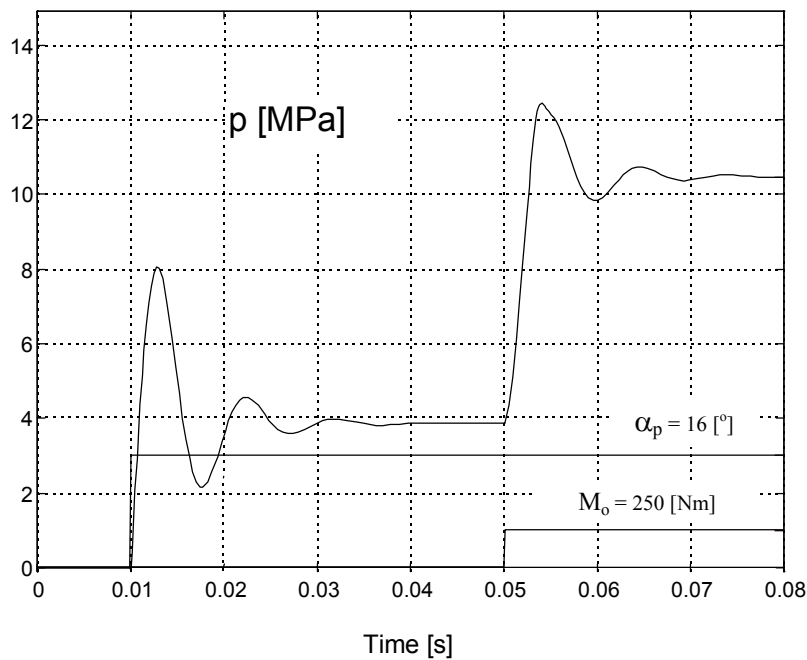


Fig. 19

- the designer is exonerated from making a program solving the system of equations of the mathematical model adopted,
- an easy making of a block diagram via a graphical editor built in the package, leading eventually to automatic making a program to solve the system of equations of the model,
- a possibility of storing the models worked out by the user in the topic-related libraries so as to be used afterwards,
- a relatively short simulation time due to rapid and effective calculating algorithms built in the package,
- a possibility of an easy selection of the method for solving the differential equations and determining the basic simulation parameters, like: final time, minimum step, maximum step and exactitude,
- a possibility of visualisation of the calculation results and their recording for further applications,
- a relatively easy change of the coefficients of the mathematical model, enabling an examination of the effect of the construction-and-operational parameters upon the properties of the systems examined.

The results shown and the concisely characterised advantages of the MATLAB-Simulink package make this package a very effective tool for performing simulation tests upon hydrostatic systems.

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