

Improving the effectivity of work with Rosin-Rammler diagram by using MATLAB[®] GUI tool

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A simple, yet powerfull tool for plotting cumulative percent oversize against particle size while using combination of a logarithmic and double logarithmic axis (RR diagram) is here proposed. It has direct support for line approximation and thus RR distribution parameters determination, along with ability to calculate oversize product from particle size value and vice versa. An insight of different approaches of RR size distribution parameters calculation is here given, along with an approach of graphing RR diagram. MATLAB[®] was used as a programming language for computation and graphical user interface design. The direct result of this work is a public accessible file covered by the BSD License.

Keywords: Rosin-Rammler diagram, size distribution, MATLAB, modelling, Weibull distribution

Introduction

Graphical plotting of sieving test results along with their tabelation and analytical expression is always advised in order to asses their full significance. Many methods of graphical presentation have therefore been developed over time, mostly plotting of the relation of cumulative undersize (or oversize) over particle size. Various combinations of axis scales can be used for this purpose (linear, logarithmic, double-logarithmic), each having its advantages and disadvantages in particular application (Wills and Napier-Munn 2006). Most of the current applications for calculations and graphing, such as spreadsheet editors are able to display data on logarithmic scale, but lack the direct support of special, double logarithmic (Rosin-Rammler) scale. As this can lead to preferring other methods of plotting even in cases where Rosin-Rammler (RR) diagram could be the best choice, a simple tool developed for actual plotting of the RR diagram would be useful.

Importance of equation solving software in determination of the parameters of the Rosin-Rammler distribution that was described many years ago (Peleg 1996) along with the lack of advanced graphing abilities of widely used spreadsheet editors stated before led to search of more advanced mathematical programming language for this particular use, with the result of MATLAB[®] being chosen.

MATLAB[®] is a powerfull, high-level programming language and interactive environment. It can be used for virtually any application in many fields of scientific and engineering activities. It has been previously sucessfully used in a field of mineral processing, ranging from application of statistical methods (Hart et al. 2006), estimation and prediction (Lin et al. 2008; Jorjani et al. 2009) up to dynamic simulation and optimization (Liu and Spencer 2004; Farzanegan and Vahidipour 2009). Powerfull built-in mathematical computation and equation solving capabilities along with customizable graphical presentation and possibility of creating graphical user interfaces (GUI) predestined MATLAB[®] to be a excelent choice for programming the tool for plotting the RR diagram and modelling the RR distribution.

Theory

Rosin-Rammler Distribution

Although the formula proposed by Rosin and Rammler (1933) (displayed as Eq. 1) was originaly derived for powdered coal, it has been shown that it can be applied to many other materials (Allen 1997). It is well suited for powder materials prepared by comminution - different types of grinding, crushing and milling. Weibull (1951) has shown wide applicability of statistical distribution function based on work published by Rosin and Rammler (1933) in many fields of human activity other than particle size distribution.

$$R(d) = 100 \cdot e^{-\left(\frac{d}{d'}\right)^n} \quad (1)$$

In Eq. 1, $R(d)$ is a cumulative percent of material retained, d is a mesh size or particle size, d' is a mean particle size (size parameter) and n is a measure of the spread of particle sizes (distribution parameter) (Djamarani and Clark

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1997; Gupta and Yan 2006). Although the RR distribution parameters d' and n can be estimated by equation fitting software, it is often done via linear regression of data graphically represented in RR diagrams.

Rosin-Rammler Diagram

After taking the logarithm of Eq. 1 twice and rearranging it gives Eq. 2:

$$\log\left(\log\frac{100}{R(d)}\right) = n.\log d - n.\log d' + \log(\log e) \quad (2)$$

Because d' and n are constants for the material given, expression displayed as Eq. 3, where e is euler number is a constant as well.

$$\log(\log e) - n.\log d' = c \quad (3)$$

Combining Eq. 2 and Eq. 3 gives Eq. 4:

$$\log\left(\log\frac{100}{R(d)}\right) = n.\log d + c \quad (4)$$

Thus a plot of double logarithm of $100/R(d)$ versus logarithm of d should give a straight line. Diagram with such an combination of axis scales is called the RR diagram (Gupta and Yan 2006; Leško 1985).

Comparison of a linear versus logarithmic scale is displayed in Fig. 1. Advantage of a logarithmic scale over a linear scale is that it does not suffer from congestions of data points in a region of finer aperture sizes (Wills and Napier-Munn 2006).

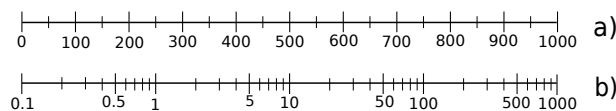


Fig. 1: Comparison of linear (a) and logarithmic (b) X-axis (Wills and Napier-Munn 2006).

Comparison of three different scales, that can be used as Y-axis in size distribution plots is shown in Fig. 2. Considering the behaviour of double logarithm of $100/R(d)$ it is clear, that such an axis can only contain values ranging from 0 to 100%, boundaries excluding. Mostly, only the values between 0.1 and 99.9% are being shown, as in the case of Fig. 2 (c).

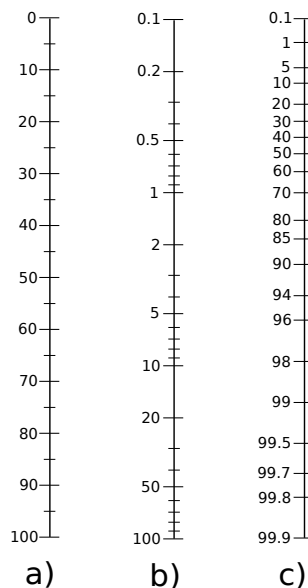


Fig. 2: Comparison of linear (a), logarithmic (b) and double logarithmic (Rosin-Rammler) (c) Y-axis (Wills and Napier-Munn 2006).

Although logarithmic Y-axis scale can be used for plotting on log-log (Gates Gaudin Schuhman) plot with straight line approximation over a wide size range, it has an disadvantage of severely contracting the region above 50% cumulative oversize and especially the region above 75% (Wills and Napier-Munn 2006). The major advantage of the

combination of logarithmic and double logarithmic axes used in RR diagram is that the approximation of size distribution data points should give a straight line over the whole region. From the line approximation, both the RR size distribution parameters that completely describes the size distribution can be obtained.

The size parameter d' can be determined by classifying given material on a mesh size $d = d'$. This substitution in Eq. 1 will produce a constant of about 36.79% material retained. The size parameter is therefore a particle size at which the approximated line crosses the value of 36.79% $R(d)$. The higher the d' value, the coarser the material. The distribution parameter n can be calculated using formula given as Eq. 5, where $[d_1, R(d_1)]$ and $[d_2, R(d_2)]$ are random collinear data points of the approximation. The lower the distribution parameter value the wider the size distribution of the material (Leško 1985).

$$n = \frac{\log\left(\log\frac{100}{R(d_2)}\right) - \log\left(\log\frac{100}{R(d_1)}\right)}{\log d_2 - \log d_1} \quad (5)$$

Results and discussion

The whole idea of graphing RR diagram in MATLAB® is based on a simple approach of adding custom axis tick marks and tick labels using the `XTick`, `Ytick`, `XTickLabel` and `YTickLabel` properties to linear plot of $\log[\log(100/R(d))]$ over $\log(d)$ as can be seen in Fig. 3.

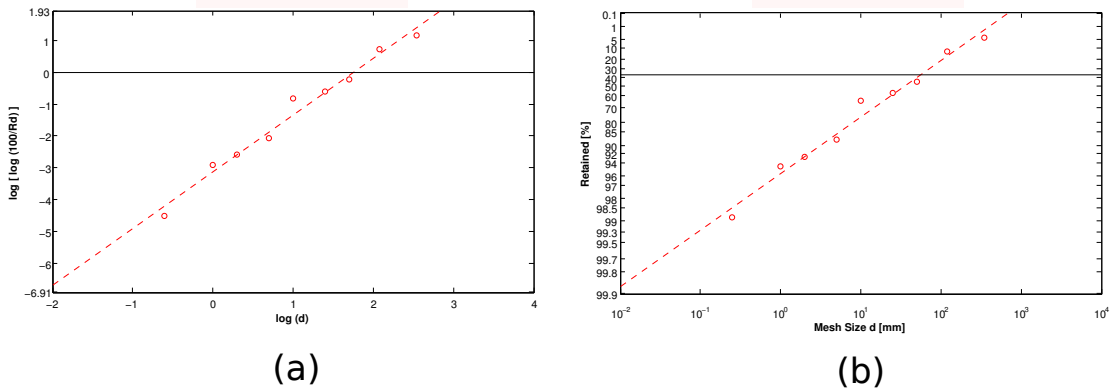


Fig. 3: RR diagram creation: (a) plain graph (b) after custom `XTick`, `Ytick` and `XTickLabel`, `YTickLabel` properties were applied

At least two different approaches of calculating RR distribution parameters are possible. These methods are summarized as diagrams shown in Fig. 4.

First approach (Fig. 4 a) is based on calculating logarithm of d and double logarithm of $100/R(d)$ in first place. The $[x, y]$ values calculated can then be used to plot the RR diagram using linear axes. Linear fitting of $y = kx + b$ equation type in least-square sense is done using the first degree `polyfit` function, which is a built-in MATLAB® function. Two random data pairs of $[x, y]$ must be calculated from the fitted function. As these are already logarithms of d and double logarithms of $R(d)$, Eq. 5 can be simplified to a form of Eq. 6 in order for the distribution parameter n to be calculated.

$$n = \frac{y_2 - y_1}{x_2 - x_1} \quad (6)$$

Corresponding x value of the size parameter d' can be calculated from linear approximation of type $y = kx + b$ with the condition of $R(d) = 36.79\%$ and thus in linear space of $y = kx + b$ equation type calculated as Eq. 7.

$$d' = 10^{\frac{y_{36.79} - b}{k}} = 10^{\frac{\log_{10}(\log_{10}(100/36.79)) - b}{k}} \quad (7)$$

Second approach (Fig. 4 b) uses MATLAB® nonlinear equation fitting routine (`lsqcurvefit` function) to fit Eq. 1 to actual data set of $[d, R(d)]$ in least square sense, while directly obtaining values of d' and n . Two random data pairs of $[d, R(d)]$ are then chosen to represent the linear fitting in the RR diagram graphed as in the previous case as double logarithm of $100/R(d)$ over logarithm of d .

In both cases, in further calculations of d or $R(d)$, only the d' and n values are used. Percent of material retained at given mesh size is calculated using Eq. 1. Calculation of mesh sizes for certain values of percent material retained is done using Eq. 8.

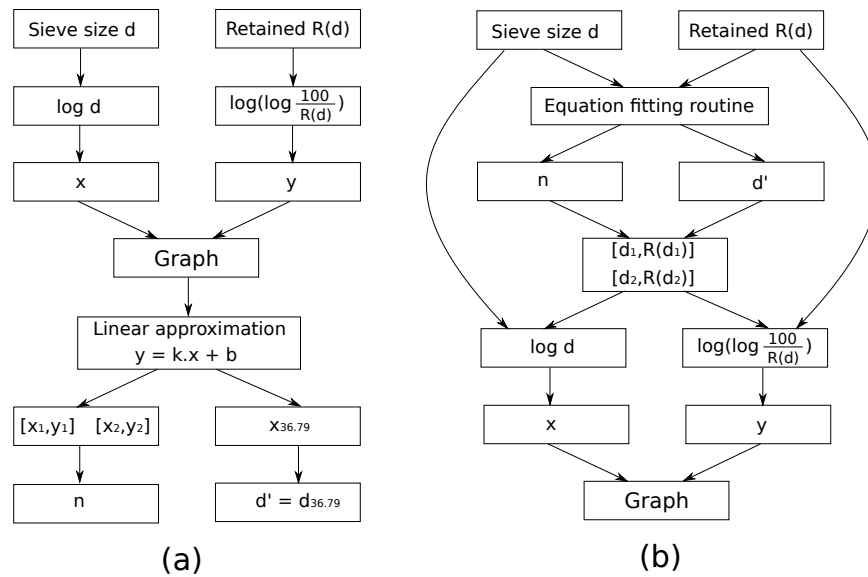


Fig. 4: Comparison of two different approaches for calculating RR distribution parameters: (a) linear approximation, (b) equation fitting

$$d = -d' \cdot \ln(R(d)/100)^{1/n} \tag{8}$$

An improvement in graphing RR diagram as described above was implemented in final tool in order to achieve better behaviour of X-axis tick marks. This was done by using default MATLAB[®] logarithmic axis after linear approximation is done. Changing the limits of this axis will thus make use of advanced algorithms for automatically adjusting displayed tick marks.

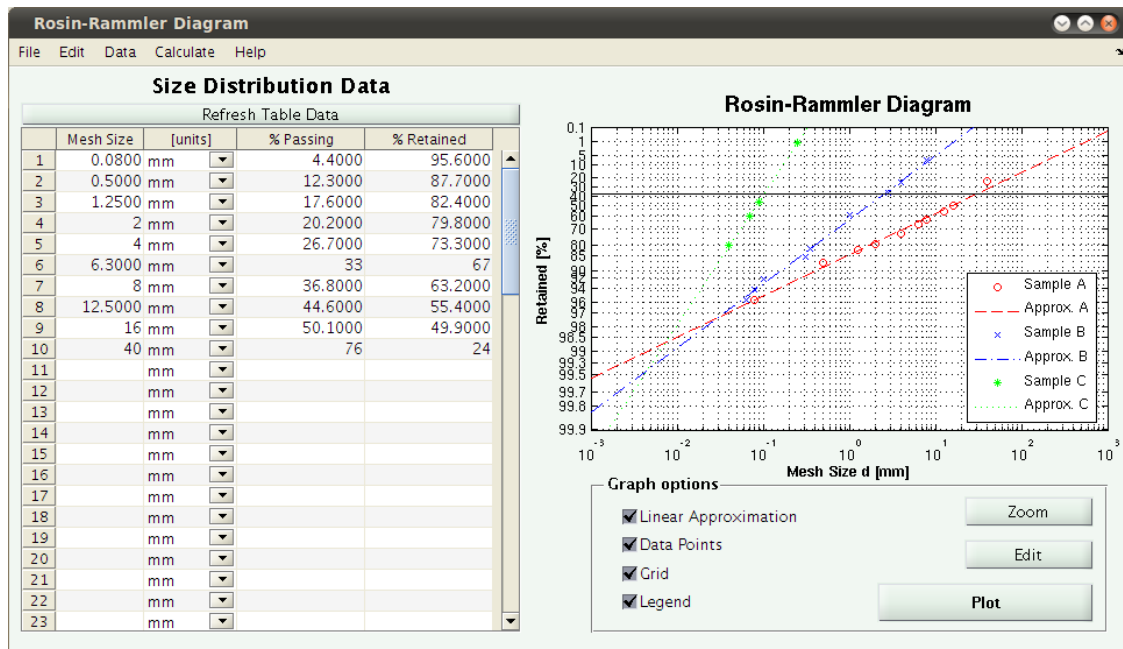


Fig. 5: Main window of the graphical user interface tool for plotting Rosin-Rammler Diagram

Based on principles and equations described above, MATLAB[®] graphical user interface tool was programmed (Fig. 5). The final file created is available online covered by the BSD license and the download location is referenced as Brezání and Zelenák (2010). Along with the file itself, function manual in form of a published M-file (html format) named "MATLAB[®] tool for plotting Rosin-Rammler size distribution diagram" can be found at this location as well. As the basic instructions of how to work with the tool are covered by the manual, these will not be discussed any further in this article.

Linear approximation vs. Equation fitting

Difference between the two approaches of determining Rosin-Rammler distribution parameters can be shown on an example of particle size distribution of different talc-magnesite ore samples in flotation processing. All three samples originate from Mútník deposit in Slovakia. *Sample A* is a feed for flotation process with high talc content, *Sample B* is a feed for flotation process with higher content of magnesite and *Sample C* is a flotation tailings with high portion of talc resulting from *Sample A* flotation. Prior to flotation process the samples were comminuted by different levels of crushing and milling. Results of laboratory sieve analysis - particle size distributions of the samples are shown in Tab. 1.

Tab. 1: Particle size distribution of powdered talc-magnesite ore in flotation

Sample A		Sample B		Sample C	
Mesh size	Retained	Mesh size	Retained	Mesh Size	Retained
[mm]	[%]	[mm]	[%]	[mm]	[%]
0.08	95.61	0.063	95.59	0.04	80.28
0.50	87.71	0.08	94.15	0.071	59.68
1.25	82.45	0.10	92.00	0.09	46.56
2.00	79.78	0.315	85.63	0.25	1.16
4.00	73.28	0.355	82.22		
6.30	66.98	1.00	58.44		
8.00	63.21	2.80	36.67		
12.50	55.40	4.00	25.04		
16.00	49.91	8.00	7.07		
40.00	23.95				

RR distribution parameters determined by both methods, as well as corresponding correlation coefficients (R) calculated between original values of material retained and values calculated using Eq. 1 are shown in Tab. 2.

Tab. 2: Comparison of RR distribution parameters determined by linear approximation and equation fitting.

		Sample A	Sample B	Sample C
Linear approx.	d' [mm]	30.77	2.56	0.1034
	n	0.556	0.841	1.639
	R	0.9909	0.9980	0.9994
Equation fit.	d' [mm]	27.06	2.59	0.1034
	n	0.594	0.799	1.594
	R	0.9937	0.9982	0.9998

It is clear, that both methods give different results - values of the RR distribution parameters. Although both of the methods produced good fits (characterized by high values of correlation coefficient R), in all three example cases equation fitting gave better results. This is caused by a disadvantage of least-square in linear space where the difference in y values counts as the same in the whole region despite the fact that the same y difference in mid region, where the $R(d)$ values are contracted can be a difference of up to tenths percent, while in the marginal areas the same value can correspond to only decimals.

Although, the second approach described seems to be more accurate in determining Rosin-Rammler size distribution parameters (higher correlation coefficient values achieved), additional MATLAB® toolbox, (Optimization Toolbox™) of which the `lsqcurvefit` function is a part of would be required to run properly. Therefore the first approach of determining RR distribution parameters is used as default in the final tool published. However, there is an option to use equation fitting routine instead, if the mentioned toolbox is found in MATLAB® cache.

Conclusion

A simple approach of graphing Rosin-Rammler (RR) diagram and a comparison of two different approaches for calculating RR distribution parameters are presented. The result file created for plotting Rosin-Rammler Diagram can be found under the reference Brezáni and Zeleňák (2010). It has been shown, that it has good capabilities for both - plotting RR diagram as well as calculating RR distribution parameters.

This tool can be of assistance in research and practical evaluation of size distribution analysis results to determine whether the RR size distribution equation is applicable for that particular material, to instantly calculate RR size

distribution parameters and to calculate percent of material retained from mesh size and vice versa. Another advantage of this interactive user interface tool is ability to save customized RR diagram in different file formats or print.

Based on the examples of size distribution data presented it is claimed, that equation fitting routine is superior to linear approximation for Rosin-Rammler distribution parameters determination in terms of higher correlation coefficients between original values of material retained and values calculated using Rosin-Rammler size distribution equation.

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