Application of back propagation artificial neural networks for gravity field modelling

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Geodesy provides a unique framework for the monitoring, understanding and prognosis of the Earth system as a whole, globally as well as locally. This understanding of geodesy is based on the three pillars: (i) geokinematics, (ii) Earth rotation, and (iii) gravity field. The third pillar of geodesy refers to the knowledge of the geometry of the gravity field of the Earth. The gravity field of the Earth addresses the problems of the transformation of geodetic observations made in physical space (affected by gravity) into geometrical space in which positions are usually defined. In addition, the shapes of equipotential surfaces and plumb lines are needed for projects involving the physical environment (e.g., flow of water). In this paper, the utility of Back Propagation Artificial Neural Network (BPANN) more widely applied in diverse fields of science among all other neural network models is investigated as an alternative tool for gravity field modelling. In order to evaluate the performance of BPANN, the gravity values are also calculated by global geopotential models (EGM2008 and EIGEN-6C4). The results are compared in terms of the root mean square error (RMSE) over a study area. It was concluded that the employment of BPANN can be a feasible gravity calculation tool for the geodetic application.

Keywords: *back propagation artificial neural networks, gravity on the Earth's surface, geopotential model, EGM2008, EIGEN-6C4.*

Introduction

Geodesy is the science of determining - representation the geometry, gravity field, and rotation of the Earth and their evolution in a 3D time-varying space. This understanding of modern geodesy is based on the definition of the three pillars of geodesy: (1) geokinematics, (2) Earth rotation, and (3) gravity field (Plag *et al.* 2009). The third pillar of this characterization of geodesy refers to the knowledge of the geometry of the gravity field of the Earth. The gravity field of the Earth addresses the problems of the transformation of geodetic observations made in physical space (affected by gravity) into geometrical space in which positions are usually defined. In addition, the shapes of equipotential surfaces and plumb lines are needed for projects involving the physical environment (e.g., flow of water). Consequently, geodesy also monitors the variability of the gravity field (Dehant 2005). Besides geodesy, the knowledge about the Earth's gravity field essentially supports research activities in geophysics and geology. At regional scales, gravity data are useful in determining the shape of the Earth, in accounting for the orbits of satellites, determining the Earth's mass and moment of inertia, and conducting geophysical mapping and interpretation of lithospheric structure and geodynamic processes. In local studies of the upper crust, gravity data can effectively address a broad range of basic geologic questions, delineate geologic features related to natural hazards (faults, volcanoes, landslides), and aid in the search for natural resources (groundwater, oil, gas, minerals, geothermal energy) (Hildenbrand *et al.* 2002).

Until a global geodetic datum is fully and formally accepted, used, and implemented worldwide, global geodetic applications require three different surfaces to be clearly defined: (i) the irregular topographic surface (the landmass topography as well as the ocean bathymetry), (ii) a geometric or mathematical reference surface called the ellipsoid, (iii) the geoid, the equipotential surface coinciding with mean sea level at ocean (Li and Götze 2001).

Gravity has an inseparable connection with these three surfaces. Accurate gravity data are the foundation for the determination of "heights". Geodesists calculate the height of locations on the Earth's surface based on the mean sea level. So knowing how gravity changes, sea level helps geodesists make more accurate measurements. Gravity corrections and gravity anomalies have been traditionally defined with respect to the height.

Due to space-based techniques, in particular, the widespread use of Global Navigation Satellite Systems (GNSS) for determining fast and accurate ellipsoidal heights (referenced to a reference ellipsoid) have incited the need for a similarly fast and accurate determination of orthometric heights related to the geoid (physically meaningful surface). Ellipsoidal heights cannot be used to determine where water will flow, and therefore are not used in topographic/floodplain mapping. Orthometric heights have a very strong correlation (>99%) with the direction of water flow and are more useful (and are colloquially-although not quite appropriately referred to by the more common term "height above sea level"). The relation between the ellipsoidal and orthometric heights is the height of the geoid above the reference ellipsoid, is usually called the geoid undulation. The geoid is the

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viable parameter for transforming ellipsoidal heights to orthometric heights and vice versa, and geoid modelling can only be done with measurements of the acceleration of gravity near the Earth's surface (Smith 2007).

The value of gravity measured at a point on the Earth's surface and the gravity calculated for the same location taking into account altitude, latitude, and topographical irregularities do not usually agree. The difference between the two is called gravity anomaly, and it represents the downward of the force of attraction produced by the underground distribution of masses differing from those of the theoretical model (Gret and Klingelé 1998). In geodesy, the gravity anomaly (Δg) is defined as the scalar difference between the Earth's gravity on the geoid (g_P) and normal gravity on the surface of the reference ellipsoid (γ) at the observation latitude.

$$\Delta g = g_P - \gamma \tag{1}$$

Geodesy requires gravity anomalies to be given on the geoid for the solution of the boundary value problem of physical geodesy, which is used to determine the figure of the Earth (Featherstone and Dentith 1997).

The artificial neural network (ANN) has been applied in diverse fields of science and engineering. ANN employments in geophysical gravity problems have increased in the last decade such as: determining depth of subsurface cavities from microgravity data (Eslam *et al.* 2001), forward modelling of gravity anomaly (Osman *et al.* 2006), sedimentary thickness variation (Zaher *et al.* 2009), 2D inverse modelling of residual gravity anomalies (Abedi *et al.* 2010), and evaluation of gravity data in a geothermal area (Kaftan *et al.* 2011).

The objective of this paper is to evaluate the utility of ANN for calculating the gravity value as an alternative calculation tool for the geodetic applications. There are numerous kinds of neural networks. However, back propagation artificial neural networks (BPANN) that have been more widely applied among all other ANN applications is used for the gravity field modelling in this paper. In order to assess the performance of BPANN, global geopotential models (GGMs) that are a representation of the earth gravity field are also used for calculating the gravity values, and the results are compared regarding the root mean square error (RMSE) over a study area.

Theoretical concepts

The feed-forward and supervised learning ANN type, BPANN (Rumelhart *et al.* 1987) was used in the artificial neural approach of this paper. In the GGM approach, Earth Gravitational Model 2008 (EGM2008) (Pavlis *et al.* 2008) and European Improved Gravity model of the Earth by New techniques 2014 (EIGEN-6C4) (Förste *et al*. 2014) were used. The detailed theoretical information about these models is given below.

Back propagation artificial neural networks

BPANN is a widely used and effective multilayer perceptron (MLP) model due to their simple implementation and flexibility. BPANN architecture consists of (i) an input layer with K neurons representing input variables to the problem, (ii) one or more hidden layers containing q neurons to help capture the nonlinearity in the data and (iii) an output layer with n neurons representing the dependent variables (Fig. 1).

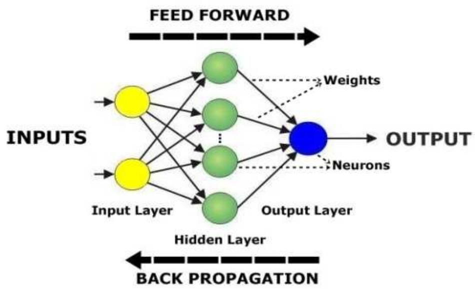

Fig. 1. The simple architecture of BPANN.

All inter-neuron connections have been associated using synaptic weights that are adjusted by an iterative back propagation algorithm known as the training process. After the training procedure, an activation function is applied to all neurons to generate the output information within a permissible amplitude range (Leandro and Santos 2007). The output of BPANN with a single output neuron (output layer represented by only one neuron, i.e. n = 1) can be expressed by:

$$y = f\left(\sum_{j=1}^{q} W_j\, f\left(\sum_{l=1}^{K} w_{j,l} x_l + w_{j,0}\right) + W_0\right) \tag{2}$$

where W_j is the weight between the *j-th* hidden neuron and the output neuron, $w_{j,l}$ is the weight between the *l-th* input neuron and the *j-th* hidden neuron, x_l is the *l-th* input parameter, $w_{j,0}$ is the weight between a fixed input equal to 1 and *j-th* hidden neuron and W_o is the weight between a fixed input equal to 1 and the output neuron (Valach *et al.* 2007).

The sigmoid function is the most commonly used activation functions satisfying the approximation conditions of BPANN and is represented by (Beale *et al.* 2010):

$$f(z) = 1 / (1 + e^{-z}) \tag{3}$$

where z is the input information of the neuron and the Euler's number, e, is the mathematical constant that is the base of the natural logarithm. The input and output values of BPANN have to be scaled in the range of $f(z) \in [0, 1]$. The back propagation algorithm based on squared error minimization corresponds to an adjustment of the weights between the hidden layer and the output layer.

BPANN design and optimisation

In this paper, BPANN is proposed according to the design and optimisation strategy followed by Yilmaz and Gullu (2014). The detailed information can be found in the relevant source and references therein. The parameters of BPANN of this paper are given in Table 1.

Tab. 1. The design and optimization parameters of BPANN.

Parameters	Settings
Training algorithm	Gradient descent
Activation function	Sigmoid
Input-Hidden-Output neurons	3-19-1
Early stopping	Test data set
Data pre-processing	Min-max normalization
Initial weight range	[-0,25; 0,25]
Learning rate (LR)	0,3
LR decrease - increase	0,5 - 1,05
Momentum term	0,6
Performance function	Mean square error

Earth Gravitational Model 2008

EGM2008 is a spherical harmonic model of the earth's external gravitational potential to degree and order 2159, with additional spherical harmonic coefficients extending up to degree 2190 and order 2159. EGM2008 is primarily developed in ellipsoidal harmonics to degree and order 2160 and transformed to spherical harmonics. EGM2008 is developed by the least squares combination of the ITG-GRACE03S gravitational model and its associated error covariance matrix, with the gravitational information obtained from a global set of area-mean free-air gravity anomalies defined on a 5′ × 5′ grid. This grid was formed by merging terrestrial, altimetry-derived, and airborne gravity data. Over areas where only lower resolution gravity data were available, their spectral content was supplemented with gravitational information implied by the topography. Over areas covered with high-quality gravity data, the discrepancies between EGM2008 geoid undulations and independent GPS/Levelling values are on the order of ±5 to ±10 cm. EGM2008 represents a milestone and a new paradigm in global gravity field modelling, by demonstrating for the first time ever, that given accurate and detailed gravimetric data, a single global model may satisfy the requirements of a very wide range of applications (Pavlis *et al.* 2012).

European Improved Gravity model of the Earth by New techniques 2014

The combined gravity field model EIGEN-6C4 is the latest combined global gravity field model up to degree and order 2190. EIGEN-6C4 is a combination of LAGEOS, GRACE RL03 GRGS, GOCE-SGG (November 2009 till October 2013) data plus 2′ × 2′ gravimetry and altimetry surface data (altimetry over the oceans, EGM2008 over continents). The combination of these different satellite and surface data sets has been done by a band-limited combination of normal equations, which are generated as a function of their resolution and accuracy (Förste *et al*. 2014).

Study area, source data, evaluation methodology

Arizona, California, Nevada, and Utah states, located in the Pacific Southwest region of the United States, are selected as the study area for the gravity calculations (Fig. 2). The study area is limited by the geographical boundaries: $31,0^{\circ} \leq \varphi \leq 41,5^{\circ}$ N; $237,5^{\circ} \leq \lambda \leq 251,0^{\circ}$ W with a rough (and mountainous) topography (Fig. 3).

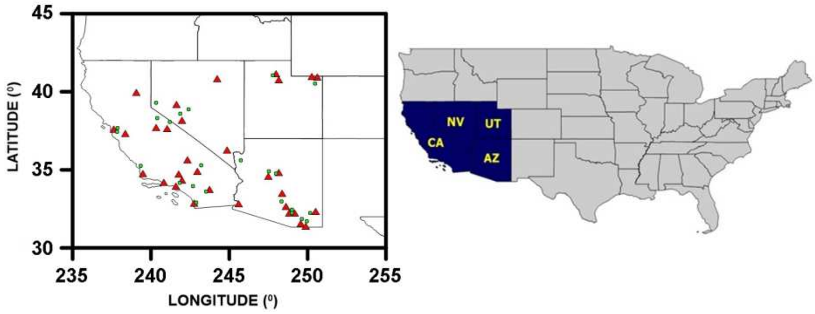

Fig. 2. The study area and the geographical point distribution (Δ; reference, o; test).

The gravity field modelling refers to a source dataset in the study area that comprises 56 gravity points (stations) belonging to the GeoNet gravity database (http://gis.utep.edu/PACES.html) compiled by the U.S. Geological Survey, the National Geospatial-Intelligence Agency (formerly the National Image and Mapping Agency), National Oceanic and Atmospheric Administration, industry and academic colleagues. The default horizontal datum is North American Datum 1983-NAD83 (WGS84), and the default vertical datum is National Geodetic Vertical Datum of 1929-NGVD29 height above mean sea level on topographical maps. Observed gravities are tied to the International Standardization Net 1971 (IGSN71) (Morelli *et al.* 1974). The IGSN71 values include the Honkasalo correction (Honkasalo 1964) for tidal deformation (Hildenbrand *et al.* 2002).

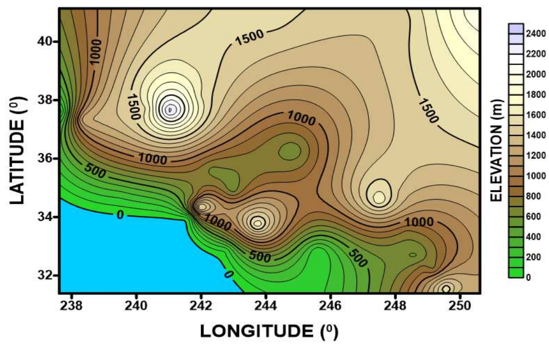

Fig. 3. The topography of the study area.

The source dataset (56 GeoNet points with observed gravity value) is classified into two groups as a reference dataset (32 points) for the training (modelling) process and a test dataset (24 points) for the accuracy assessment. The reference points are selected to cover the study area from outside, and the validation points are selected as densification points of the network formed by the reference points. The geographical distribution of the reference and test points within the study area is plotted in Fig. 2 and the statistical values of these datasets are given in Table 2.

Tab. 2. The statistics of the datasets (units in mgal)

	Reference	Test
Minimum	979004,928	979862,547
Maximum	979974,574	979993,683
Mean	979511,607	979449,305
Std. Dev.	234,433	257,063

The evaluation of gravity field modelling is focused on the residuals between the observed gravity and the gravity calculated by BPANN, EGM2008, and EIGEN-6C4:

$$\text{Residual } g = g_{observed} - g_{calculated} \qquad (4)$$

For the statistical analysis of gravity residuals, the statistics (minimum, maximum, and mean) were determined and investigated by RMSE because RMSEs are sensitive to even small errors to measure the deviations between known and calculated discharges on ANNs (Gullu *et al.* 2011). RMSE is defined by:

$$RMSE = \sqrt{\frac{1}{t}\sum_{i=1}^{t}(\text{Residual } g_i)^2} \quad (5)$$

where t is the number of the test points.

Case study

The reference points are used to train BPANN, and the test points are used to evaluate the performance of BPANN in ANN approach. In GGM approach, the reference points are used to generate the gravity field model of the study area. The accuracies of ANN and GGM approaches are assessed by using the test points.

For the case study, BPANN with a single hidden layer is proposed with three neurons in the input layer and one neuron in the output layer. The geographical coordinates (φ, λ) and orthometric height (H) of the point are selected as input quantities, and the gravity (g) of the point (on the Earth's surface) is used as output quantity for training and testing procedure of BPANN. A trial-and-error strategy was employed in order to determine the optimal number of the neurons in the hidden layer of BPANN, and, the optimal number of neurons in the hidden layer was selected as 19 for BPANN by a MATLAB ANN module that allows changing the parameters of BPANN dynamically. BPANN is trained by using the gravity values of the reference points. After the training procedure, the gravity values of the test points are calculated by using the trained BPANN.

In GGM approach, the gravity field of the study area is generated from the reference dataset by Surfer 12 surface modelling software. The gravity is defined as the magnitude of the gradient of the potential (including the centrifugal potential) at a given point. The gravity (on the Earth's surface) is calculated by the following equations:

$$W = W_a + \Phi \quad (6)$$

where W, is the potential associated with the rotating Earth; W_a, is the attraction potential; and Φ, is the centrifugal potential.

$$g = |\nabla \mathrm{W}| \quad (7)$$

where g, is the gradient of the potential W.

$$|\nabla W| = \sqrt{[W_{ar} + \Phi_r]^2 + \left[\frac{1}{r\cos\varphi}(W_{a\lambda} + \Phi_\lambda)\right]^2 + \left[\frac{1}{r}(W_{a\varphi} + \Phi_\varphi)\right]^2} \quad (8)$$

The derivatives of Eq. (8) in spherical harmonics are:

$$\begin{aligned}
W_{ar} &= -\frac{GM}{r^2}\sum_{l=0}^{l_{\max}}\left(\frac{R}{r}\right)^l (l+1)\sum_{m=0}^{l} P_{lm}(\sin\varphi)(C_{lm}^W \cos m\lambda + S_{lm}^W \sin m\lambda) \\
W_{a\lambda} &= \frac{GM}{r}\sum_{l=0}^{l_{\max}}\left(\frac{R}{r}\right)^l \sum_{m=0}^{l} m\, P_{lm}(\sin\varphi)(S_{lm}^W \cos m\lambda - C_{lm}^W \sin m\lambda) \\
W_{a\varphi} &= \frac{GM}{r}\sum_{l=0}^{l_{\max}}\left(\frac{R}{r}\right)^l \sum_{m=0}^{l} \frac{\partial P_{lm}(\sin\varphi)}{\partial\varphi}(C_{lm}^W \cos m\lambda + S_{lm}^W \sin m\lambda)
\end{aligned} \quad (9)$$

The derivatives of centrifugal potential are:

$$\Phi_r = \omega^2 r(\cos\varphi)^2;\ \ \Phi_\lambda = 0;\ \ \Phi_\varphi = -\omega^2 \mathrm{r}^2 \cos\varphi \sin\varphi \quad (10)$$

The notations are: (r, φ, λ), spherical geocentric coordinates of computation point (radius, longitude, latitude); GM, product of the gravitational constant and the mass of the Earth; R, reference radius; l, m, degree, order of spherical harmonic; P_{lm}, fully normalised Lengendre functions; $C^W{}_{lm}$, $S^W{}_{lm}$, Stokes' coefficients of the disturbing potential (fully normalised); ω, angular velocity of the Earth (Barthelmes 2013). The gravity, $|\nabla W|$, is calculated from Eq.s (9) and (10).

The gravity values of the test points were computed from this (reference) gravity field. The gravity values based on GGMs are interpolated from the closest grid points by Kriging method using software and coefficients obtained from International Centre for Global Earth Models (ICGEM) (http://icgem.gfz-potsdam.de/ICGEM).

Results and conclusions

The statistical values of the gravity residuals associated with the test data set are presented in Table 3. The model representations have been adopted for the comparative evaluation of BPANN and GGMs by producing residual maps. The gravity residual maps of the test points associated with BPANN, EGM2008 and EIGEN-6C4 are given in Figures 4, 5, and 6, respectively.

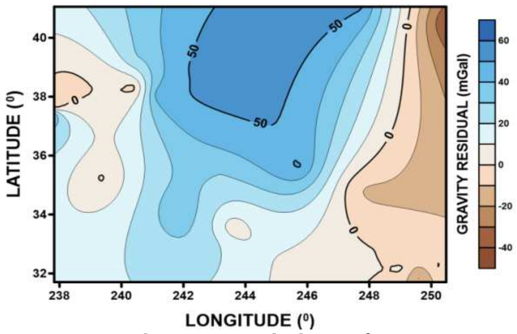

Fig. 4. The gravity residual map of BPANN

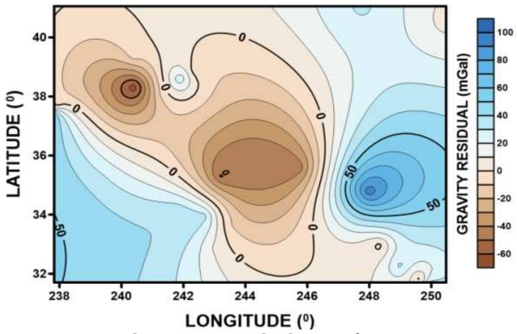

Fig. 5. The gravity residual map of EGM2008

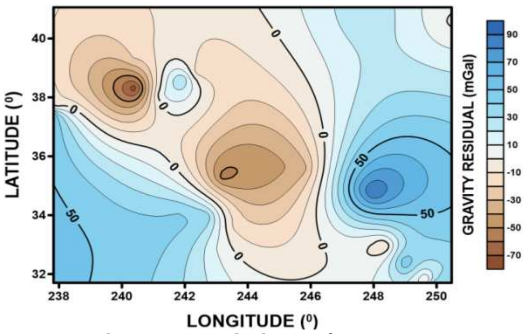

Fig. 6. The gravity residual map of EIGEN-6C4

When the results summarized in Table 3 are evaluated, it can be seen from Figures 4, 5, and 6 that BPANN calculated the point gravities more accurately in the study area, with respect to EGM2008 and EIGEN-6C4, in terms of RMSE. BPANN has an increasing gravity residual sequence over areas where only poor gravity data were available. Whereas, EGM2008 and EIGEN-6C4, similarly, have an increasing gravity residual sequence at sea, and at the mountainous areas (H >1000 m.)

Tab. 3. The statistics of the gravity residuals (units in mgal.).

	BPANN	EGM2008	EIGEN-6C4
Min.	-40,691	-66,700	-76,624
Max.	52,692	96,547	91,653
Mean	13,701	12,601	13,668
RMSE	30,047	38,981	41,135

- The objective of this paper was to evaluate the utility of ANN for the gravity field modelling for the geodetic applications. Based on the qualitative and quantitative results of this paper, the following conclusions can be drawn:
- (1) ANN can be considered as a feasible gravity calculation tool for the geodetic applications. BPANN calculated the gravity with a better accuracy (in terms of RMSE) when it is compared to GGMs, because of its model-free estimation.

- (2) EGM2008 has better statistics than EIGEN-6C4. EGM2008 can be used as a reference earth geopotential model for a further gravity calculation at regional and national scales in the USA.
- (3) With more dense gravity stations and with improved geographical coverage, more accurate gravity field modelling can be expected from BPANN and also GGMs.
- (4) The combination of diverse ANNs (e.g., different training algorithms and activation functions, additional hidden layers and neurons) as a trend surface approximator with GGMs would be an appealing tool for gravity field modelling, because of ANN's adaptive 'learning by example' feature.
- For ANN applications, there is no need to incorporate any assumptions about the frequency distribution of the data (i.e., the normal distribution of the data in geodetic problems). Besides, ANN can always be updated with new training data to obtain better results. In this regard, ANN outstands from GGMs. Despite the feasibility of ANN for gravity calculation, improving extrapolation ability, and dealing with uncertainty should receive further attention in the future studies.

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