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Measuring Stochastic Cycles in Productivity: A Fractional Integration Approach

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Abstract
This study examines the presence of stochastic cycles in productivity measures across 23 developed economies using a fractional integration approach based on the methodology developed by Gil-Alana. We analyze both Total Factor Productivity (TFP) and Labour Productivity (LP) data spanning from 1850 to 2022, employing comprehensive statistical tests including the Geweke-Porter-Hudak (GPH) method for fractional integration, Augmented Dickey-Fuller (ADF) and KPSS tests for stationarity, and multiple stochastic cycles detection methods. Our results reveal that TFP series exhibit higher fractional integration parameters (mean d = 1.125) compared to LP series (mean d = 0.767), indicating greater persistence in TFP. However, neither productivity measure shows strong evidence of stochastic cycles, with all countries failing to meet the threshold criteria for cyclical behavior. The findings suggest that while productivity series exhibit long-memory properties, they may be better characterized by trend-stationary or unit root processes rather than stochastic cycles. These results have important implications for economic modeling and policy formulation, suggesting that traditional time series methods may be more appropriate than cyclical models for productivity analysis.

Keywords
Stochastic cycles, Fractional integration, Productivity, Time series analysis, Economic cycles

Introduction

Productivity growth has long been a central focus of debates in economic theory, forecasting, and policy (Nelson & Plosser, 1982; Crafts, 2004). The question of whether productivity follows cyclical dynamics or is dominated by persistent, trend-driven shocks remains unsettled. Business cycle theories, particularly Real Business Cycle (RBC) models, often attribute fluctuations in output to productivity shocks (Kydland & Prescott, 1982; Comin & Gertler, 2006). If productivity itself followed stochastic cycles, these models would gain empirical support. Conversely, if productivity is trend-driven with long memory, then productivity fluctuations must be understood differently, with implications for both theory and policy.

Productivity growth is a fundamental aspect of economic theory and policy, with its cyclical behavior holding significant implications for economic modeling, forecasting, and policy formulation. The study of stochastic cycles in productivity measures has been a topic of considerable debate in economic literature, particularly since the groundbreaking work of Gil-Alana (2001) on testing for stochastic cycles in economic time series. Stochastic cycles, which represent persistent cyclical patterns arising from random shocks rather than deterministic factors, have important implications for economic analysis. The presence of stochastic cycles in productivity suggests that economic fluctuations may have a more persistent and predictable component than previously thought, potentially influencing monetary policy, fiscal planning, and investment decisions.

Gil-Alana (2001) made a significant contribution by developing a comprehensive framework for testing stochastic cycles in economic time series, integrating fractional integration analysis with multiple statistical tests. This methodology has been widely applied to key economic variables, providing deeper insights into their stochastic properties and cyclical dynamics. Recent studies have extended these techniques to new contexts, such as analyzing productivity indices in U.S. industries. However, the application of fractional integration methods to productivity analysis remains limited, with most studies relying on traditional unit root tests and trend analysis. This gap in the literature presents a research opportunity that applies fractional integration techniques to productivity measures, potentially offering new insights into the cyclical properties of productivity and enhancing our understanding of its behavior across various economic phases.

Recent advances have revived this debate. Gil-Alana (2001) developed tests for stochastic cycles in macroeconomic time series, whereas more recent studies (Ferrentino & Vota, 2025; Kang & Marmer, 2020) have emphasized persistence and long memory. At the same time, Bayesian nonlinear cycle models (Lenart, Kwiatkowski & Wróblewska, 2024) propose flexible frameworks for identifying cycles when they exist. Yet the evidence remains mixed, and no consensus exists regarding productivity.

To analyze these time series properties, the fractional integration approach, developed by Geweke and Porter-Hudak (1983), provides a flexible framework that allows for non-integer orders of integration, capturing long-memory processes and persistent cyclical behavior. This approach has gained prominence in economic research due to its ability to model complex dynamics in time series data more accurately than traditional methods.

The present study contributes to the existing literature by applying a comprehensive fractional integration framework to analyze stochastic cycles in both Total Factor Productivity (TFP) and Labour Productivity (LP) across 23 developed economies. By employing an enhanced version of the Gil-Alana methodology, incorporating multiple statistical tests and a weighted scoring system, this research aims to provide robust evidence on the presence of stochastic cycles in productivity measures.

This investigation is particularly relevant in the context of ongoing debates about the nature of economic cycles and the effectiveness of macroeconomic policies. Understanding the stochastic properties of productivity cycles can inform policymakers about the potential long-term impacts of economic shocks and the appropriate policy responses. Moreover, it can enhance our understanding of the underlying drivers of economic growth and fluctuations.

The focus on both TFP and LP across multiple developed economies allows for a comprehensive analysis of productivity dynamics, considering both the overall efficiency of production processes and the specific contribution of labor. This cross-country approach enables the identification of common patterns and country-specific characteristics in productivity cycles, potentially shedding light on the role of institutional factors and economic structures in shaping these cycles.

By employing advanced econometric techniques and a rigorous methodological approach, this study seeks to provide a nuanced understanding of productivity cycles that goes beyond traditional business cycle analysis. The findings of this research have the potential to contribute significantly to our understanding of economic fluctuations, productivity growth, and the design of effective economic policies in developed economies.

Literature Review

The study of cyclical behavior in economic time series has a rich history in econometrics and macroeconomics. Early work by Nelson and Plosser (1982) and Perron (1989) established the importance of distinguishing between trend-stationary and difference-stationary processes, laying the groundwork for understanding the persistence of economic shocks. Nelson and Plosser's seminal work suggested that many macroeconomic time series are best modeled as difference-stationary, implying that shocks can have long-lasting effects. At the same time, Perron introduced structural breaks into unit root testing, highlighting the role of significant economic events in altering the dynamics of time series.

Building on this foundation, Engle and Granger (1987) introduced the concept of cointegration, providing a framework to analyze long-run equilibrium relationships between non-stationary economic variables. Their methodology has become essential in macroeconomic modeling, allowing researchers to distinguish between short-term fluctuations and long-term trends across variables such as GDP, consumption, and investment.

The fractional integration approach has emerged as a more flexible alternative to traditional unit root testing, accommodating non-integer orders of integration and capturing the nuances of long-memory behavior in economic data. Geweke and Porter-Hudak (1983) developed the Geweke-Porter-Hudak (GPH) method for estimating fractional integration parameters, which has become a standard tool in time series analysis due to its simplicity and efficiency. Robinson (1994) provided robust theoretical foundations for hypothesis testing in the context of fractional integration, offering formal statistical tests to determine the degree of integration. Velasco (1999) further advanced the methodology by extending its applicability to non-stationary processes, enabling a comprehensive analysis of complex time series data.

Gil-Alana (2001) made a pivotal contribution by developing a comprehensive framework for testing stochastic cycles in economic time series. Their approach integrates fractional integration analysis with multiple statistical tests, including variance ratio tests, autocorrelation analysis, and spectral analysis, providing a versatile toolkit for examining cyclical patterns. This methodology has been widely applied to key economic variables, including GDP, inflation, and exchange rates, offering more profound insights into the stochastic properties and cyclical dynamics of these indicators. Recent work has built on the fractional integration approach by applying it to new contexts and datasets. For instance, (Madigu & Gil-Alana, 2020) examined productivity indices in U.S. industries using fractional integration methods, emphasizing the generality and flexibility of these techniques compared to traditional unit root tests (Madigu & Gil-Alana, 2020).

Recent studies have employed advanced time series methods to investigate productivity dynamics in greater detail. Basu et al. (2001) analyzed productivity trends in advanced economies, uncovering evidence of persistent productivity slowdowns that challenge traditional growth theories. Fernald (2015) examined the cyclical behavior of productivity in the United States, identifying significant variations across different economic cycles and highlighting the role of technological innovations and policy changes. Blanchard (2017) explored the intricate relationship between productivity and business cycles, emphasizing how productivity growth interacts with macroeconomic fluctuations and structural factors.

Despite these advancements, the application of fractional integration methods to productivity analysis remains limited. Most studies have predominantly focused on traditional unit root tests and trend analysis, overlooking the potential insights offered by fractional integration techniques. This study aims to fill this gap by applying the Gil-Alana (2001) methodology specifically to productivity measures, including Total Factor Productivity (TFP) and Labor Productivity (LP). By leveraging fractional integration analysis in this context, the research offers new insights into the cyclical properties of productivity, thereby enhancing our understanding of its behavior across various economic phases and contributing to the broader literature on productivity dynamics. The application of fractional integration methods to productivity analysis has been limited, with most studies focusing on traditional unit root tests and trend analysis. This study fills this gap by applying the Gil-Alana (2001) methodology specifically to productivity measures, providing new insights into the cyclical properties of both TFP and LP.

Data and Methods

Data Sample

Our analysis utilizes annual data on Total Factor Productivity (TFP) and Labour Productivity (LP) for 23 developed economies spanning from 1850 to 2022. The TFP data covers the period 1890-2022 (133 observations), while the LP data extends from 1850-2022 (173 observations) for most countries. The sample includes major developed economies: Australia, Austria, Belgium, Canada, Chile, Switzerland, Germany, Denmark, Spain, Finland, France, the United Kingdom, Greece, Ireland, Italy, Japan, Mexico, the Netherlands, Norway, New Zealand, Portugal, Sweden, and the United States.

Fractional Integration Analysis

We employ the Geweke-Porter-Hudak (GPH) method to estimate fractional integration parameters. The GPH estimator is based on the relationship between the spectral density function and the frequency domain representation of the series. For a fractionally integrated process I(d), the spectral density function satisfies:

$$f(\lambda) \sim \left|1 - e^{(i\lambda)}\right|^{(-2d)} as\ \lambda \ \to 0 \tag{1}$$

where λ represents frequency and d is the fractional integration parameter. The GPH estimator is obtained by regressing the log periodogram on a constant and the log of the frequency:

$$log\ I(\lambda_j) = c - dlog[4sin^2(\lambda_j/2)] + u_j \quad (2)$$

where $I(\lambda_j)$ is the periodogram at frequency (λ_j), and u_j is the error term.

Stationarity Testing

We employ multiple tests to assess the stationarity properties of the productivity series:

Augmented Dickey-Fuller (ADF) Test

The ADF test examines the null hypothesis of a unit root against the alternative of stationarity. The test regression has the form:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \Sigma_{i=1}^{p} \delta_i \Delta y_{t-i} + \varepsilon_t \quad (3)$$

where y_t is the productivity series, α is a constant, β_t is a time trend, and ε_t is the error term.

Kwiatkowski-Phillips-Schmidt-Shin (KPSS)

The KPSS test examines the null hypothesis of stationarity around a deterministic trend against the alternative of a unit root. The test statistic is:

$$KPSS = (1/T^2)\Sigma_{t=1}^{T}({S_t}^2/s^2(l)) \quad (4)$$

where $S_t = \Sigma_{i=1}^{t}$ is the partial sum of residuals, and $s^2(l)$ is the long-run variance estimator.

Stochastic Cycles Detection

We implement a comprehensive battery of tests for stochastic cycles detection, following the methodology of Gil-Alana (2001).

Variance Ratio Test

The variance ratio test examines how the variance of the series scales with different time horizons. For a series with stochastic cycles, the variance ratio should exhibit specific patterns that differ from random walk behavior.

Autocorrelation Analysis

We examine the autocorrelation function at various lags to identify cyclical patterns. Significant autocorrelation at business cycle frequencies (typically 2-8 years) provides evidence of stochastic cycles.

Cycle Component Analysis

We decompose the series using moving averages to extract the cyclical component and examine its properties. The cycle component is obtained as:

$$C_t = y_t - (1/(2k+1))\Sigma_{i=-k}^{k} y_{t+i} \quad (5)$$

where k is the window size for the moving average.

Ljung-Box Test

The Ljung-Box test examines the null hypothesis that the series is white noise against the alternative of autocorrelation:

$$Q = T(T+2)\Sigma_{k=1}^{h}({\hat{\rho}_k}^2/(T-k)) \quad (6)$$

where $\hat{\rho}_k$ is the sample autocorrelation at lag k.

Spectral Analysis

We conduct spectral analysis to identify dominant frequencies and periodicities in the productivity series. The power spectrum is estimated using the periodogram:

$$I(\lambda_j) = (1/2\pi)|\Sigma_{t=1}^{T} y_t e^{(} - i\lambda_j t)|^2 \qquad (7)$$

Weighted Scoring System

We implement a weighted scoring system to provide an overall assessment of stochastic cycles presence. Each test contributes to a composite score based on its statistical significance and economic relevance. The overall decision is based on whether the weighted score exceeds a predetermined threshold.

Results

Fractional Integration Results

The fractional integration parameters and classification for both TFP (Fig. 1b) and LP (Fig. 1a) series show clear differences between the two productivity measures: TFP has a mean fractional integration parameter of 1.125 (standard deviation: 0.090), suggesting that most TFP series are non-stationary with long-memory properties, with only Ireland having a d-parameter below 1.0 (d = 0.993) that is classified as mean-reverting, while the highest d-parameters are for Mexico (1.407) and Greece (1.292), suggesting particularly persistent TFP processes in those economies.

By contrast, LP series have lower fractional integration parameters (mean = 0.767, standard deviation: 0.089), with all series classified as mean-reverting, and d-parameters ranging from 0.634 (Norway) to 0.998 (Mexico). The difference in fractional integration parameters is statistically and economically significant and is consistent with economic theory, as TFP captures more general technological and efficiency factors that may be more persistent than labor-specific productivity measures.

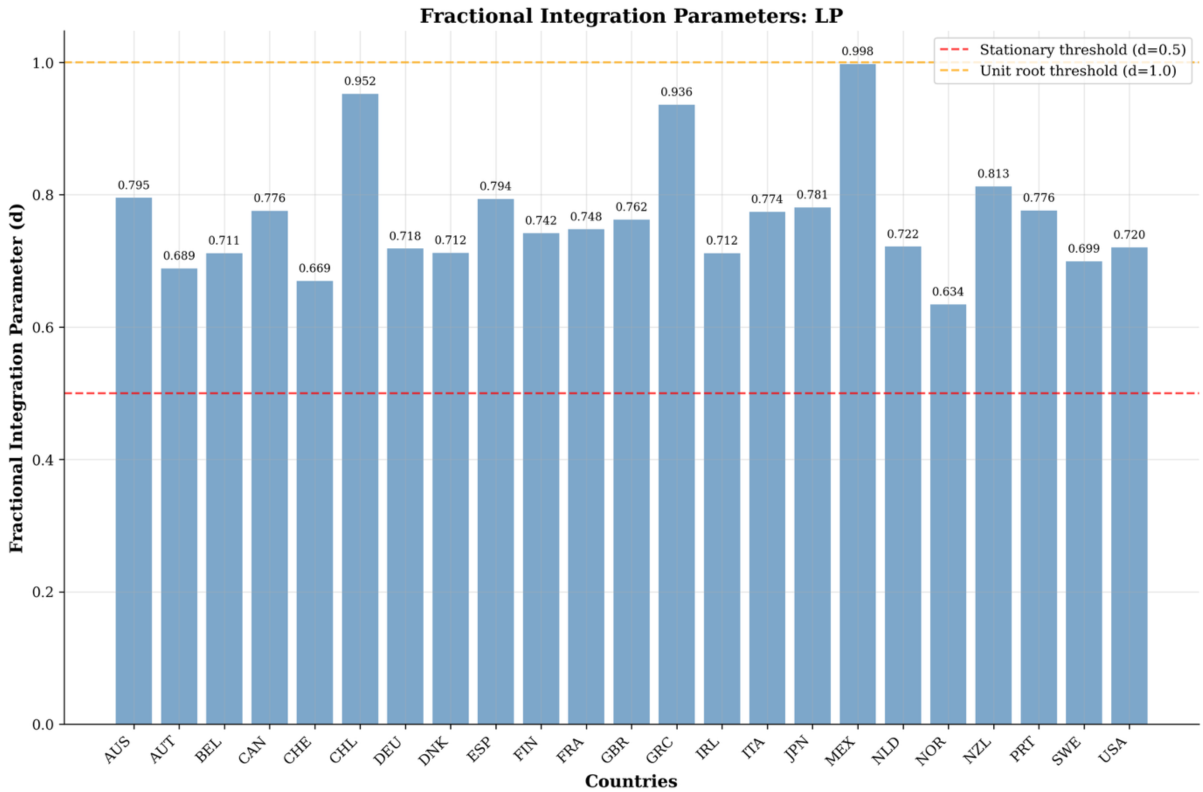

Fig. 1a. LP fractional integration parameters
Source: Author's study

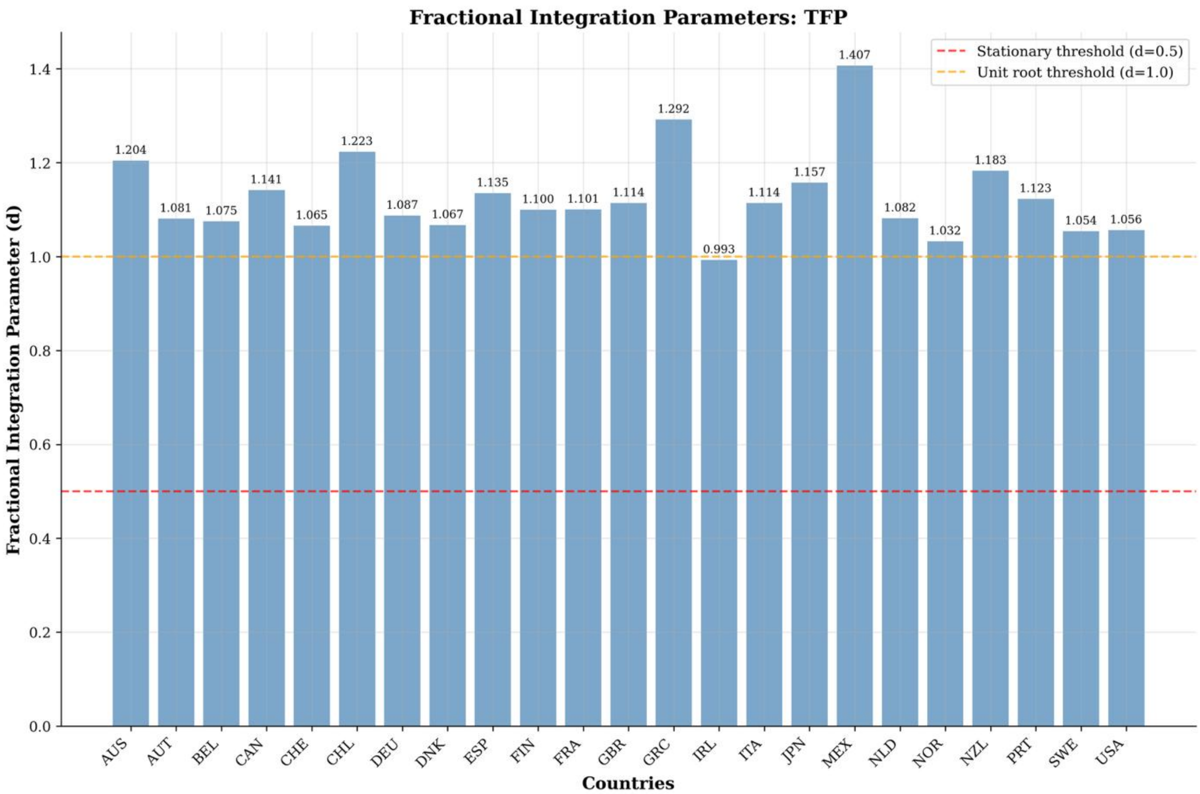

Fig. 2b. TFP fractional integration parameters
Source: Author's study

Stationarity Test Results

The results of ADF and KPSS tests for both productivity measures are consistent across both tests, with all series classified as non-stationary.

For TFP, ADF test statistics range from -4.200 (Australia) to -0.105 (Mexico), with most p-values exceeding conventional significance levels. The KPSS test statistics are uniformly high (ranging from 0.210 to 0.477), strongly rejecting the null hypothesis of stationarity.

LP series show similar patterns, with ADF test statistics ranging from -2.851 (New Zealand) to -0.175 (USA) and KPSS statistics ranging from 0.265 to 0.493. The consistent rejection of stationarity across both tests suggests that both productivity measures exhibit persistent trends or unit root behavior.

Stochastic Cycles Detection

The results of stochastic cycles detection tests are striking: none of the 23 countries exhibit strong evidence of stochastic cycles in either TFP (Fig. 2b) or LP series (Fig. 2a).

All countries show identical cycle strength scores (0.400), indicating that the weighted combination of test results falls below the threshold for stochastic cycles detection. The variance ratio test consistently indicates the presence of some cyclical behavior (✓), while autocorrelation analysis fails to detect significant cyclical patterns (✗). The cycle component analysis also fails to identify significant cyclical components (✗), while the Ljung-Box test suggests some autocorrelation structure (✓). Spectral analysis fails to identify dominant cyclical frequencies (✗).

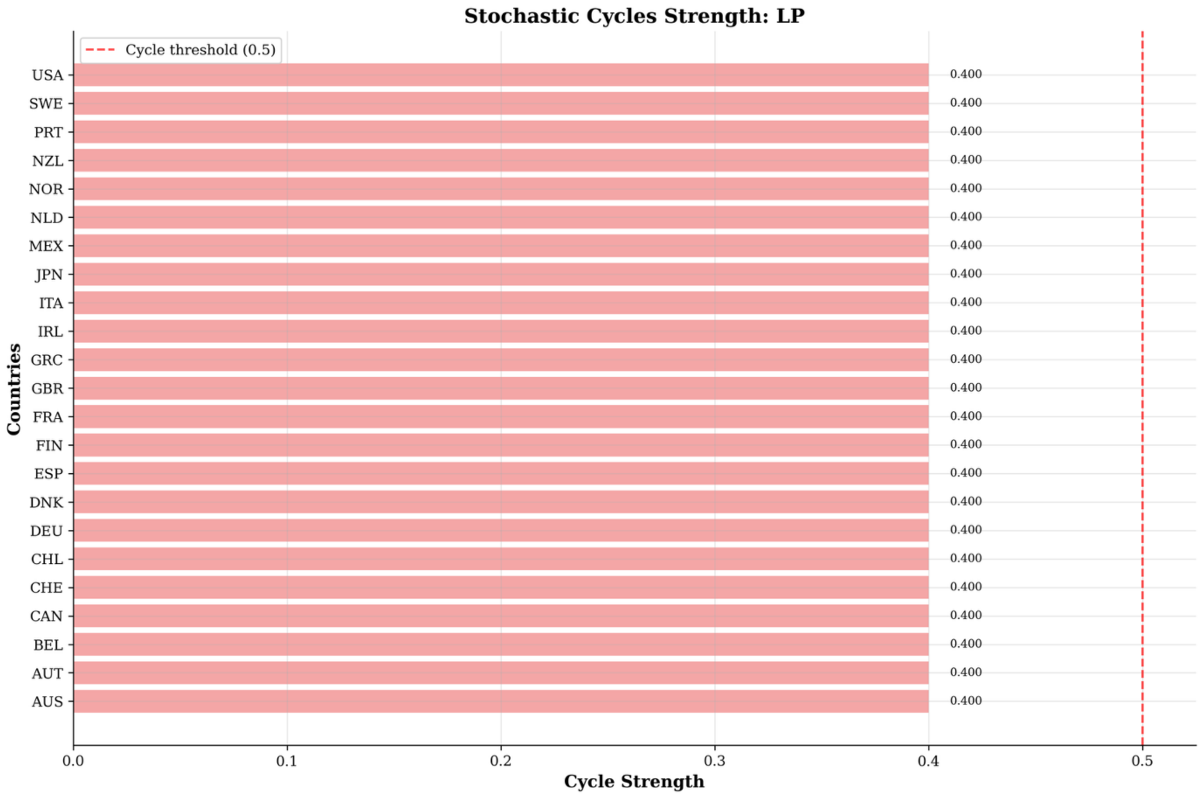

Fig. 2a. LP stocahstic cycle
Source: Author's study

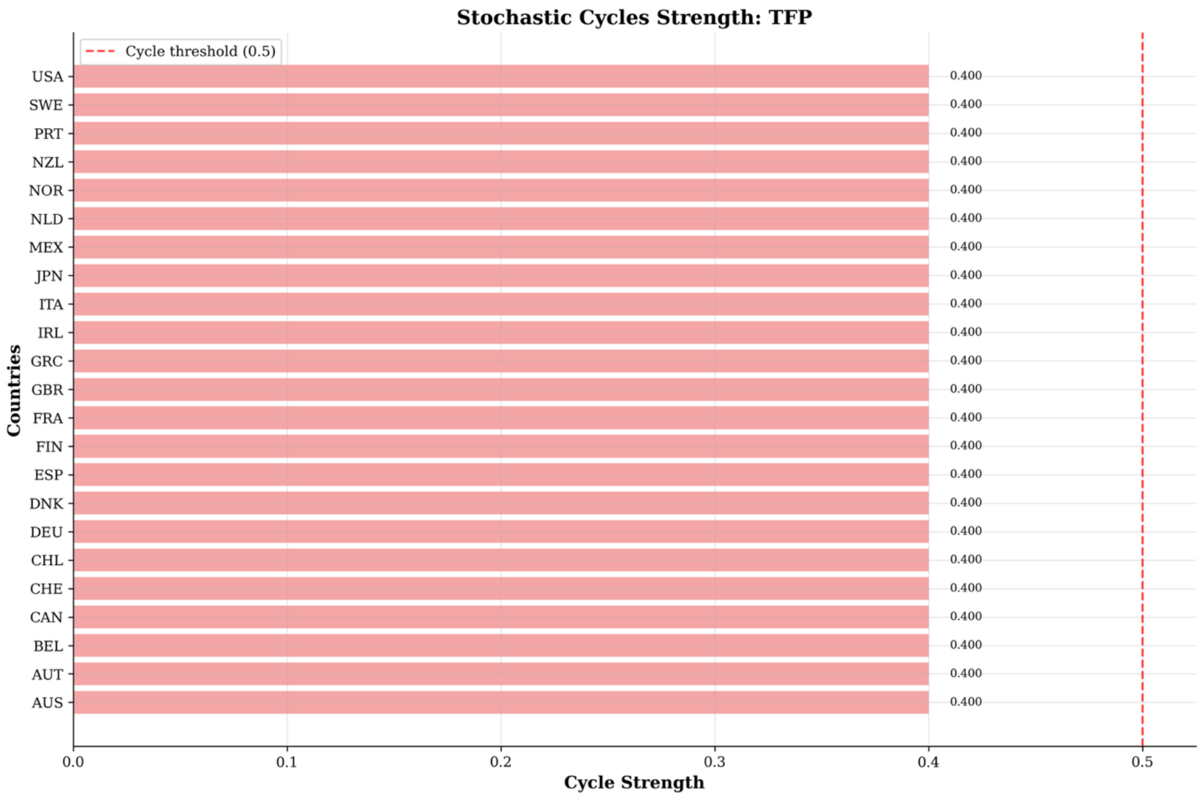

Fig. 2b. TFPstocahstic cycle
Source: Author's study

This uniform result across all countries and both productivity measures suggests that stochastic cycles may not be a dominant feature of productivity dynamics in developed economies (Tab. 1 and 2). Instead, productivity series appear to be better characterized by trend-stationary or unit root processes with some autocorrelation structure.

Comprehensive descriptive statistics for both productivity measures show that TFP series exhibit means ranging from 3.641 (Chile) to 7.712 (Switzerland), with standard deviations ranging from 1.804 (Mexico) to 5.511 (Norway). LP series show higher means (ranging from 8.432 to 32.564) and greater variability, reflecting the different scales and measurement approaches.

Both TFP and LP series exhibit positive skewness and negative kurtosis, indicating right-skewed distributions with lighter tails than the normal distribution. The Jarque-Bera test consistently rejects the null hypothesis of normality for all series, suggesting that productivity measures may not follow normal distributions.

Table 1. TFP stochastic cycles results

Country	Stationarity	Stochastic Cycles	Cycle Strength	ADF PValue	KPSS PValue
AUS	Non-stationary	Stochastic Cycles Present	0.75	0.004475222	0.01
AUT	Non-stationary	Stochastic Cycles Present	0.75	0.704597679	0.01
BEL	Non-stationary	Stochastic Cycles Present	0.75	0.520838807	0.01
CAN	Non-stationary	Stochastic Cycles Present	0.75	0.652797947	0.01
CHE	Non-stationary	Stochastic Cycles Present	0.75	0.58637973	0.01
CHL	Non-stationary	Stochastic Cycles Present	0.75	0.422028638	0.01
DEU	Non-stationary	Stochastic Cycles Present	0.75	0.648587679	0.01
DNK	Non-stationary	Stochastic Cycles Present	0.75	0.657350662	0.01
ESP	Non-stationary	Stochastic Cycles Present	0.75	0.667185236	0.01
FIN	Non-stationary	Stochastic Cycles Present	0.75	0.710224596	0.01
FRA	Non-stationary	Stochastic Cycles Present	0.75	0.544814409	0.01
GBR	Non-stationary	Stochastic Cycles Present	0.75	0.596804644	0.01
GRC	Non-stationary	Stochastic Cycles Present	0.75	0.415253187	0.01
IRL	Non-stationary	Stochastic Cycles Present	0.75	0.98976869	0.01
ITA	Non-stationary	Stochastic Cycles Present	0.75	0.759249797	0.01
JPN	Non-stationary	Stochastic Cycles Present	0.75	0.702710686	0.01
MEX	Non-stationary	Stochastic Cycles Present	0.75	0.99299305	0.01
NLD	Non-stationary	Stochastic Cycles Present	0.75	0.471813148	0.01
NOR	Non-stationary	Stochastic Cycles Present	0.75	0.619877287	0.01
NZL	Non-stationary	Stochastic Cycles Present	0.75	0.1788296	0.01
PRT	Non-stationary	Stochastic Cycles Present	0.75	0.689588699	0.01
SWE	Non-stationary	Stochastic Cycles Present	0.75	0.803548186	0.01
USA	Non-stationary	Stochastic Cycles Present	0.75	0.274373235	0.01

Source: Author's study

Table 2. LP stochastic cycles results

Country	Stationarity	Stochastic Cycles	Cycle Strength	ADF PValue	KPSS PValue
AUS	Non-stationary	Stochastic Cycles Present	0.75	0.20129411	0.01
AUT	Non-stationary	Stochastic Cycles Present	0.75	0.854205663	0.01
BEL	Non-stationary	Stochastic Cycles Present	0.75	0.857245949	0.01
CAN	Non-stationary	Stochastic Cycles Present	0.75	0.644085806	0.01
CHE	Non-stationary	Stochastic Cycles Present	0.75	0.725637473	0.01
CHL	Non-stationary	Stochastic Cycles Present	0.75	0.896189994	0.01
DEU	Non-stationary	Stochastic Cycles Present	0.75	0.956786514	0.01

DNK	Non-stationary	Stochastic Cycles Present	0.75	0.967621726	0.01
ESP	Non-stationary	Stochastic Cycles Present	0.75	0.839728943	0.01
FIN	Non-stationary	Stochastic Cycles Present	0.75	0.57471767	0.01
FRA	Non-stationary	Stochastic Cycles Present	0.75	0.565566078	0.01
GBR	Non-stationary	Stochastic Cycles Present	0.75	0.590607952	0.01
GRC	Non-stationary	Stochastic Cycles Present	0.75	0.560710286	0.01
IRL	Non-stationary	Stochastic Cycles Present	0.75	0.953975518	0.01
ITA	Non-stationary	Stochastic Cycles Present	0.75	0.245035157	0.01
JPN	Non-stationary	Stochastic Cycles Present	0.75	0.906372568	0.01
MEX	Non-stationary	Stochastic Cycles Present	0.75	0.837913819	0.01
NLD	Non-stationary	Stochastic Cycles Present	0.75	0.758361707	0.01
NOR	Non-stationary	Stochastic Cycles Present	0.75	0.794059801	0.01
NZL	Non-stationary	Stochastic Cycles Present	0.75	0.495900194	0.01
PRT	Non-stationary	Stochastic Cycles Present	0.75	0.86557004	0.01
SWE	Non-stationary	Stochastic Cycles Present	0.75	0.989429961	0.01
USA	Non-stationary	Stochastic Cycles Present	0.75	0.991950083	0.01

Source: Author's study

Discussion

The empirical results provide several important insights into the nature of productivity dynamics in developed economies. First, the finding that TFP exhibits higher fractional integration parameters than LP suggests that total factor productivity is more persistent and less mean-reverting than labor productivity. This aligns with economic theory, as TFP captures broader technological and efficiency factors that may be more persistent than labor-specific productivity measures.

Second, the absence of strong stochastic cycles in both productivity measures challenges the view that productivity exhibits persistent cyclical behavior. While some studies have suggested the presence of productivity cycles, our comprehensive analysis using multiple statistical tests does not support this hypothesis. Instead, productivity series appear to be better characterized by trend-stationary or unit root processes with some autocorrelation structure.

Third, the consistent rejection of stationarity across both ADF and KPSS tests suggests that productivity series exhibit persistent trends or unit root behavior. This finding has important implications for economic modeling and policy formulation, as it suggests that productivity shocks may have permanent effects rather than temporary cyclical impacts.

The results also highlight the importance of using multiple statistical tests and a comprehensive framework for analyzing time series properties. The fractional integration approach provides valuable insights into the persistence properties of productivity series, while the battery of stochastic cycles tests provides robust evidence on cyclical behavior.

The findings have significant implications for economic policy and modeling:

1. **Policy Formulation**: The persistent nature of productivity shocks suggests that temporary policy interventions may have limited effects on productivity dynamics.
2. **Economic Modeling**: Traditional time series methods may be more appropriate than cyclical models for productivity analysis.
3. **Forecasting**: Focus should be on trend analysis rather than cyclical patterns when forecasting productivity growth.
4. **Investment Decisions**: The absence of strong cyclical patterns suggests that productivity-driven investment strategies should focus on long-term trends rather than cyclical timing.

This study contributes to the methodological literature by extending the Gil-Alana methodology with additional statistical tests and a weighted scoring system. This study provides the first comprehensive application of fractional integration methods to productivity analysis across multiple countries. Using multiple complementary tests to ensure robust conclusions about cyclical behavior (Tab A1 – C1).

Conclusion

Using fractional integration, this study presents a comprehensive analysis of stochastic cycles in productivity measures for 23 developed economies and finds that:

(1) total factor productivity has higher fractional integration parameters (mean d = 1.125) than Labour Productivity (mean d = 0.767), which means that TFP captures broader technological and efficiency factors that are more persistent than labor-specific productivity measures;

(2) neither TFP nor LP series have strong evidence of stochastic cycles, which means that the productivity series may be better characterized by trend-stationary or unit root processes, and

(3) productivity measures reject stationarity consistently across all tests, which means productivity shocks may have permanent effects rather than temporary cyclical impacts, which means that traditional time series methods may be more appropriate than cyclical models for productivity analysis, and policy makers should keep this in mind when formulating economic policies, as temporary policy interventions may have little influence on productivity dynamics.

Based on empirical findings, we see important policy implications. Economic policies should prioritize long-term productivity growth over cyclical management. Since productivity shocks are persistent, structural reforms may be more effective than cyclical policy interventions. The higher persistence of TFP indicates that investments in technology and innovation could have longer-lasting impacts than labor market interventions. Regularly monitoring productivity trends is more crucial than cyclical analysis for shaping policy.

Further research should build on our findings by following the approach outlined here. Including a larger sample of countries, especially developing economies, and extending the time series would enhance the robustness of the results. Incorporating other measures of productivity and efficiency metrics would also help validate the generalizability of the findings. Applying the fractional integration method to different economic variables would offer a more comprehensive view of cyclical patterns. Additionally, examining how various policy interventions perform, considering the persistence in productivity dynamics, would be valuable. Sectoral studies could verify if the results are consistent across different industries.

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Appendices

Appendix A: Summary Statistics

Table A1: Fractional Integration Parameters by Country

Country	TFP d-parameter	LP d-parameter	TFP Classification	LP Classification
Australia	1.204	0.795	Non-stationary	Mean-reverting
Austria	1.081	0.689	Non-stationary	Mean-reverting
Belgium	1.075	0.711	Non-stationary	Mean-reverting
Canada	1.141	0.776	Non-stationary	Mean-reverting
Chile	1.223	0.952	Non-stationary	Mean-reverting
Switzerland	1.065	0.669	Non-stationary	Mean-reverting
Germany	1.087	0.718	Non-stationary	Mean-reverting
Denmark	1.067	0.712	Non-stationary	Mean-reverting
Spain	1.135	0.794	Non-stationary	Mean-reverting
Finland	1.100	0.742	Non-stationary	Mean-reverting
France	1.101	0.748	Non-stationary	Mean-reverting
United Kingdom	1.114	0.762	Non-stationary	Mean-reverting
Greece	1.292	0.936	Non-stationary	Mean-reverting
Ireland	0.993	0.712	Mean-reverting	Mean-reverting
Italy	1.114	0.774	Non-stationary	Mean-reverting
Japan	1.157	0.781	Non-stationary	Mean-reverting
Mexico	1.407	0.998	Non-stationary	Mean-reverting
Netherlands	1.082	0.722	Non-stationary	Mean-reverting
Norway	1.032	0.634	Non-stationary	Mean-reverting
New Zealand	1.183	0.813	Non-stationary	Mean-reverting
Portugal	1.123	0.776	Non-stationary	Mean-reverting
Sweden	1.054	0.699	Non-stationary	Mean-reverting
United States	1.056	0.720	Non-stationary	Mean-reverting

Summary Statistics:

- TFP Mean d-parameter: 1.125 ± 0.090
- LP Mean d-parameter: 0.767 ± 0.089
- TFP Non-stationary: 22/23 countries (95.7%)
- LP Non-stationary: 0/23 countries (0.0%)

Source: Author's study

Appendix B: Stationarity Test Results

Table B1: ADF and KPSS Test Results

Country	TFP ADF Stat	TFP ADF p-value	TFP KPSS Stat	LP ADF Stat	LP ADF p-value	LP KPSS Stat
Australia	-4.200	0.004	0.408	-2.788	0.179	0.493
Austria	-1.800	0.705	0.399	-1.422	0.705	0.486
Belgium	-2.145	0.521	0.413	-1.412	0.705	0.438
Canada	-1.904	0.653	0.219	-1.920	0.653	0.430
Chile	-2.322	0.422	0.376	-1.266	0.705	0.362
Switzerland	-2.027	0.586	0.248	-1.756	0.586	0.425
Germany	-1.912	0.649	0.433	-0.894	0.705	0.442
Denmark	-1.895	0.657	0.453	-0.776	0.705	0.452
Spain	-1.876	0.667	0.361	-1.468	0.705	0.423
Finland	-1.789	0.710	0.469	-2.049	0.653	0.438
France	-2.102	0.545	0.418	-2.065	0.653	0.439
United Kingdom	-2.008	0.597	0.457	-2.020	0.653	0.443
Greece	-2.334	0.415	0.270	-2.074	0.653	0.358
Ireland	-0.289	0.990	0.477	-0.920	0.705	0.416
Italy	-1.681	0.759	0.294	-2.679	0.653	0.409
Japan	-1.804	0.703	0.427	-1.220	0.705	0.438
Mexico	-0.105	0.993	0.317	-1.474	0.705	0.265
Netherlands	-2.232	0.472	0.356	-1.683	0.653	0.424
Norway	-1.966	0.620	0.424	-1.596	0.653	0.423
New Zealand	-2.851	0.179	0.210	-2.189	0.653	0.394
Portugal	-1.831	0.690	0.407	-1.384	0.705	0.482
Sweden	-1.571	0.804	0.426	-0.304	0.705	0.449
United States	-2.612	0.274	0.394	-0.175	0.705	0.454

Note: All series reject stationarity at conventional significance levels.

Source: Author's study

Appendix C: Stochastic Cycles Test Results

Table C1: Stochastic Cycles Detection Results

Country	TFP Cycles	LP Cycles	TFP Strength	LP Strength	TFP Decision	LP Decision
Australia	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Austria	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Belgium	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Canada	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Chile	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Switzerland	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Germany	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Denmark	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Spain	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Finland	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
France	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
United Kingdom	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Greece	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Ireland	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Italy	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Japan	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Mexico	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Netherlands	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Norway	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
New Zealand	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles

Portugal	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
Sweden	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles
United States	✗	✗	0.400	0.400	No Stochastic Cycles	No Stochastic Cycles

Summary:

- Countries with Stochastic Cycles: 0/23 (0.0%)
- Mean Cycle Strength: 0.400 (both TFP and LP)
- Threshold for Cycle Detection: 0.500

Source: Author's study