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Foreign Exchange Market Performance: Evidence from Bivariate Time Series Approach

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Abstract
In this study, we propose a novel approach to currency analysis in the foreign exchange market by utilizing Escoufier's RV coefficient to capture the similarity between currencies based on bivariate time series. Unlike traditional methods that use univariate time series of exchange rate returns, the bivariate approach accounts for both bid and ask prices, providing a more comprehensive understanding of currency relationships. By applying this method, we are able to uncover hidden patterns and structural features within the currency network that are not captured by conventional analyses. The paper also demonstrates the applicability of this technique in the context of FOREX analysis, offering a detailed example to highlight its effectiveness. This approach not only enhances the accuracy of measuring currency similarity but also improves the overall analysis of market dynamics. Ultimately, our findings suggest that the RV coefficient provides significant advantages in studying the interconnectedness of currencies in financial markets.

Keywords
Foreign exchange market, RV correlation coefficient, network analysis, minimum spanning tree.

Introduction

The global foreign exchange (FOREX) market is the world's largest and most important financial market, spanning all countries. It is completely decentralized and has the highest daily trading volume, reaching trillions of US dollars (Górski et al., 2008). The FOREX market's dynamics appear to be more complex than those of any other market (Kwapień et al., 2009). Therefore, it is essential to study and comprehend these interactions to capture its pricing mechanisms. In this regard, the correlation matrix has long been used to quantify interactions, and the information generated can be very helpful if sufficient data is provided beforehand. However, the process of extracting information from the correlation matrix is not as straightforward as it seems (Eom et al., 2007).

Understanding FOREX market dynamics requires advanced tools and methodologies due to its complexity and high-frequency trading nature. Traditional approaches often fail to capture the underlying patterns and interactions within the market, making it necessary to explore alternative techniques. Network-based methods, particularly those leveraging correlation matrices, have shown promise in modeling financial systems effectively. This study focuses on enhancing these methods through multivariate statistical tools, enabling a more comprehensive analysis of currency interactions.

We use the RV coefficient to measure similarity among multivariate time series. More specifically, we focus on measuring similarity among two-dimensional currencies based on bid and ask prices and then analyze currency behavior. For this purpose, MST will be employed to filter information contained in the complex network of the correlation matrix among currencies. As an application, 45 major world vector currencies from May 2008 to May 2013 will be studied.

This research is the first to demonstrate how the concept of Escoufier's vector correlation can appropriately measure similarity among multivariate time series in a currency network. The motivation of this article is to apply the vector correlation coefficient (RV) to define similarity among currencies, where each currency is represented by a bivariate time series, and to analyze the vector correlation network in terms of the topological structure of the currencies.

The integration of RV coefficients and MST offers several advantages. It allows for the identification of dominant currencies, detection of clusters, and measurement of network centrality based on bivariate data. Furthermore, this approach can highlight hidden patterns and structural dependencies that are not visible using traditional univariate methods. Ultimately, the findings have implications for risk management, portfolio diversification, and understanding the propagation of financial shocks within global currency markets.

Litreture Review

Research on the FOREX market has long focused on understanding the structural relationships among currencies using various analytical approaches. Traditional methods predominantly rely on univariate time series models that analyze exchange rate returns, as these approaches effectively capture price dynamics and relationships. The introduction of network analysis techniques, particularly the minimum spanning tree (MST) method proposed by Mantegna and Stanley (1999), has significantly advanced the study of complex interactions in financial markets. MST has become a key tool in filtering essential information from correlation matrices and visualizing the interconnectedness of currencies and stocks (McDonald et al., 2005; Wang et al., 2012; Górski et al., 2006).

Early studies applying MST methods focused on correlations between stock returns and currency behaviors. Notable contributions include works by McDonald et al. (2005), Brida et al. (2009), Ortega and Matesanz (2006), Mizuno et al. (2006), Naylor et al. (2007), Kwapień et al. (2009), Keskin et al. (2011), Jang et al. (2011), Wang et al. (2012, 2013), Kwapień et al. (2009), Sinha and Kovur (2014), Tumminello et al. (2007), and Bonanno et al. (2004). These studies effectively demonstrated the applicability of MST in identifying dominant nodes and clusters within financial networks. However, the majority of these studies represented currencies as univariate time series based solely on closing prices, which limited the ability to incorporate richer information, such as bid and ask price dynamics.

Most FOREX market and stock market models have focused on analyzing the behavior of closing prices, as exemplified by the works of Drożdż et al. (2007), Oh et al. (2011), Vogel and Saravia (2014), Oh et al. (2012), Sato and Hołyst (2008), and Breymann et al. (2009). However, this approach neglects the potential to integrate information from bid and ask prices, which could offer deeper insights into currency market dynamics.

Nonetheless, in finance, many researchers have used methods of multivariate statistical analysis, such as factor analysis (Song and Zhang, 2014) principal component analysis (Matteson and Tsay, 2011), and cluster analysis (Kakizawa et al., 1998). Vector correlation was first described by Hotelling in 1936. After that, the correlation coefficient between two sets of complex vectors was studied by Masuyama (1939, 1941) and Rozeboom in 1965. Subsequently, the measurement of correlation among vector variables was examined by Kshirsagar (1969), Escoufier (1973), Coxhead (1974), Cramer (1974), Popper Shaffer and Gillo (1974), Cramer and Nicewander (1979), Stephens (1979), Ramsay et al. (1984), Robert et al. (1985), and Roy and Cléroux (1993).

The majority of multivariate correlations proposed are actually functions of the canonical correlations. Escoufier (1973) and Stephens (1979) introduced a vector coefficient by assuming that two sets of vectors are perfectly correlated if there is an orthogonal transformation that can align them. Robert and Escoufier (1976) also demonstrated that the Escoufier coefficient serves as a unifying tool for some multivariate methods.

Escoufier (1973) and Robert and Escoufier (1976) introduced the use of the RV coefficient for presenting different methods of multivariate analysis. These methods were formulated as solutions to optimization problems under various constraints.

Their work demonstrated how the RV coefficient could capture structural similarity among multidimensional datasets, making it suitable for analyzing financial networks.

Recent studies have emphasized the importance of multivariate analysis in financial markets to address limitations of univariate models. Escoufier's RV coefficient has been shown to outperform traditional Pearson correlation in scenarios involving multidimensional datasets, which makes it particularly applicable for analyzing bid and ask prices in currency markets.

Despite advancements in network analysis and correlation metrics, there is limited research integrating RV coefficients with MST to analyze bid-ask price dynamics in the FOREX market. Existing studies either focus on univariate time series or overlook the bid-ask spread, which plays a crucial role in capturing market microstructures and liquidity effects. This paper addresses this gap by proposing a novel method that combines Escoufier's RV coefficient with MST to evaluate the topological structure of currency networks.

This research builds upon previous methodologies and contributes to the literature by introducing a multivariate perspective to currency analysis. It leverages the RV coefficient's ability to capture 2-dimensional relationships and MST's capability to visualize network structures, thereby offering a comprehensive framework for analyzing currency similarities and dependencies.

Material and Methods

Data of 45 major world currencies from May 5, 2008 until May 4, 2013 were downloaded from http://www.oanda.com/ for daily bid price and ask price. The foreign exchange rate among currencies is mostly stated in relation to the US dollar. In this study also the daily exchange rate of each country's currency for US dollar is used as basic data.

In order to measure the similarities and differences in the synchronous time evolution of a pair of currencies, we study the correlation between the daily logarithmic changes in the bid price and ask price of two currencies viewed in a multi-dimensional setting. Consider the change in the successive differences of the natural logarithm of price, which is defined as: $V_i = \ln Z_i\,(t+1) - \ln Z_i\,(t)$, where Z_i is the rate of currency i at time t.

Theoretically, let X be an $(n \times p)$ data matrix (of p variables $X_1, \cdots, X_p$) in $\Re^p$ and Y be an $(n \times q)$ data matrix (of q variables $Y_1, \cdots,\ Y_q$) in $\Re^q$, corresponding to two sets of variables defined for the same n observations.

A commonly used matrix correlation, which allows for a different number of variables, is the RV-coefficient introduced by Escoufier (1973):

$$RV_{X,Y} = \frac{tr(S_{XY}\,S_{YX})}{\sqrt{tr(S^2{}_{XX})tr(S^2{}_{YY})}}$$

Where X and Y represent multivariate time series of currencies (1973), S_{XX} is the sample covariance of X (resp. S_{YY} is the sample covariance of Y), and S_{XY} is cross-covariance matrix of the variables X and Y. In our study, X and Y are data matrices of bivariate time series of bid price and ask price where $p = q = 2$.

The RV coefficient has the following properties:

- For any X and Y data matrix: $0 \le RV \le 1$.
- $RV_{X,Y} = 1$ if $Y = aBX + c$, with B an orthogonal matrix, a a scalar, and c a constant vector.
- $RV_{X,Y} = 0$ where all the variables of one set are uncorrelated to all the variables of the following set; $X^tY = 0$ or $Y^tX = 0$.
- If $p = q = 1$, then $RV_{X,Y} = r^2$, the square of the univariate correlation (Pearson correlation coefficient).
- If $p = 1$and $q > 1$, then $RV_{X,Y} = R^2/\sqrt{q}$ where R^2 is the determination coefficient when we regress X with respect to Y (Escoufier, Y., 2006).

Distance d between two data matrices X and Y (two configurations) can also be defined by using the norm $\|A\| = \sqrt{tr(A^tA)}$; A is a square matrix. We define

$$d(X,Y) = \left\| \frac{X^tX}{\sqrt{tr(XX^t)^2}} - \frac{Y^tY}{\sqrt{tr(YY^t)^2}} \right\| = \left\| \frac{S_{XX}}{\sqrt{tr(S^2{}_{XX})}} - \frac{S_{YY}}{\sqrt{tr(S^2{}_{YY})}} \right\|$$

$$= \sqrt{2}\sqrt{1 - \frac{tr(S_{XY}\,S_{YX})}{\sqrt{tr(S^2{}_{XX})tr(S^2{}_{YY})}}} = \sqrt{2}\sqrt{1 - RV(X,Y)}$$

Now consider data matrix D of size $n \times n$ where each element $d(x,y)$ is a Euclidean distance since it satisfies the following three axioms of metric;

(i) $d(x,y) \geq 0$ and $d(x,y) = 0 \Leftrightarrow x = y$,

(ii) $d(x,y) = d(y,x)$, and

(iii) $d(x,y) \leq d(x,l) + d(l,y)$.

This matrix represents dissimilarities network currencies defined by its bid price and ask price. To analyze this network, MST is constructed using Kruskal's algorithm as suggested in 1956 (Kruskal, 1956). This MST is the principal tool to simplify the complex system of currencies in the form of an optimal tree. MST is helpful for understanding the network structure and properties of FOREX markets. Beneficial insights on the global structure of the financial data, particularly in the equity and exchange rate markets have been demonstrated with MST methodology (Tumminello et al., 2005).

To analyze the topological properties of currencies in MST, four major centrality measures (degree, betweeness, closeness and eigenvector centrality) as interpretation tools are examined. Centrality is a basic concept in network analysis (Borgatti, 2005). The degree of centrality for a node i refers to the number of links attached to the node i. It is defined as:

$$C_D(i) = deg(i)\,/(n-1)$$

Degree centrality describes the degree of importance of information for each node according to the idea that more important nodes are more active and therefore should have more connections. The Betweenness centrality describes the frequencies of nodes in the shortest paths between indirectly connected nodes (Freeman,1977). High centrality scores indicate that a node can reach others on relatively short paths, or that a node lies on considerable fractions of shortest paths connecting others. For node i, the betweenness centrality can be computed through the following formula:

$$C_B(i) = \frac{1}{(n-1)(n-2)/2} \sum_{j,k \in G} \frac{d_{(j,k)}(i)}{d_{(j,k)}}$$

where $d_{(j,k)}$ denotes the number of shortest paths between node j and node k, and $d_{(j,k)}(i)$ represents the number of shortest paths containing point as an intermediately in the geodesics between node i and node k.

The closeness centrality can be regarded as a measure of the time to spread information from a currency to other reachable currencies in the network, and is defined as the mean geodesic distance (i.e., the shortest path) between a node i and all of the nodes reachable from i (Sabidussi, 1966; Borgatti, 2005). Given a node i and a graph G, it can be defined as:

$$C_C(i) = \frac{(n-1)}{\sum_{j \in G} d_G(i,j)}$$

where $d_G(i,j)$ signifies the minimum distance between node i and node j. It assesses the effectiveness of one node in a graph. The nodes with higher values are closer to the others (on average) (Sabidussi, 1966; Tabak et al., 2010).

Eigenvector centrality measure is used to determine which node is connected to most connected nodes. The eigenvector centrality of node i is defined as:

$$e_i = \frac{1}{\lambda_{max}} \sum_{j=1}^{n} (A_{ij}x_j)$$

Where $x = (x_1, x_2, \cdots, x_n)^{\,t}$ is the eigenvector associated with the largest eigenvalue λ_{max} of the adjacency matrix A. It is the weighted average of the scores x_j of all vertices linked to node $\boldsymbol{i}$. The larger the value of e_i, the more influence of node i to other nodes directly or indirectly. High-scoring node is the one that has connections to other high-scoring nodes.

Method verification and results

In this section, to simplify the complex network of multi-dimensional currencies, we apply the similarity matrix based on correlation between multivariate time series of currencies to intuitively understand the network structure. MST tool is applied for 45 currencies. MST is used for filtering networks of financial market that resulting in simpler forms of network that can facilitate the analysis of currency exchange markets (Tabak et al., 2010). The MST chooses the $n-1$ stronger (i.e. shorter) links which span all the currencies by using Kruskal's algorithm.

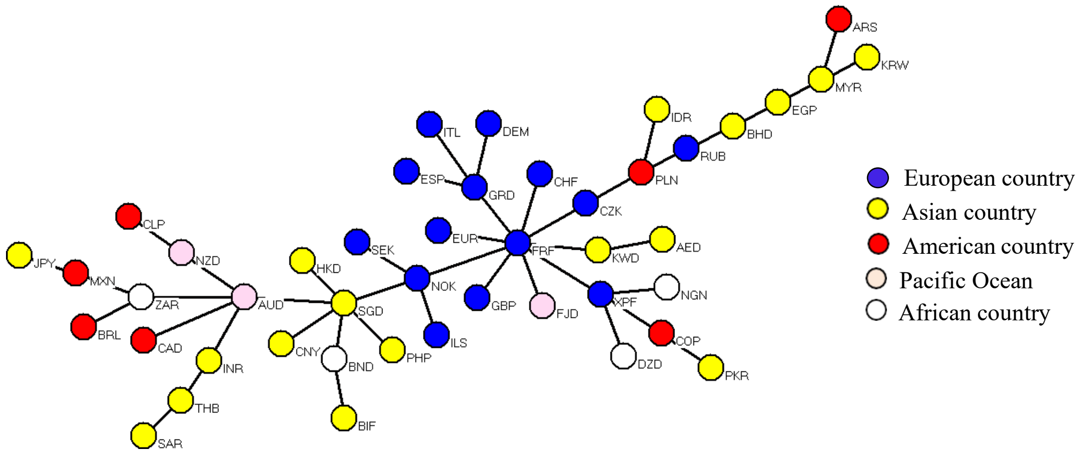

Fig. 1. MST based on RV coefficienthe

Figure1 shows several notable results in currency network. We find that most of European corresponding countries are linked together; other currencies are distributed sporadically. FRF (French- Francs) with highest links (9) is in central position of MST while EUR that is a major global currency exist in the lowest linkage. FRF has the highest score in terms of all measures of centralities (degree, betweeness, closeness and eigenvectors). See Table1 for these measures.

Tab. 1. Countries, respective symbols and centrality measures

No.	Country	Currency	Degree	Closeness	Betweenness	Eigenvectors
1	Algeria	DZD	0.023	0.206	0	0.081
2	Argentina	ARS	0.023	0.116	0	0.000
3	Australia	AUD	0.114	0.251	0.396	0.110
4	Bahamas	BHD	0.045	0.169	0.169	0.010
5	Brazil	BRL	0.023	0.173	0	0.013
6	British	GBP	0.023	0.246	0	0.189
7	Brunei	BND	0.045	0.228	0.045	0.071
8	Burundi	BIF	0.023	0.186	0	0.022
9	Canada	CAD	0.023	0.202	0	0.034
10	France	XPF	0.091	0.257	0.174	0.266
11	Chili	CLP	0.023	0.170	0	0.011
12	China	CNY	0.023	0.226	0	0.064
13	Colombia	COP	0.045	0.208	0.045	0.090
14	Czech	CZK	0.045	0.270	0.304	0.214
15	Egypt	EGP	0.045	0.148	0.130	0.003
16	Europe	EUR	0.023	0.246	0	0.189
17	Fiji	FJD	0.023	0.246	0	0.189

18	France	FRF	0.205	0.324	0.743	0.617
19	Germany	DEM	0.023	0.204	0	0.080
20	Greece	GRD	0.091	0.254	0.133	0.262
21	Hong Kong	HKD	0.023	0.226	0	0.064
22	India	INR	0.045	0.206	0.089	0.037
23	Indonesia	IDR	0.023	0.187	0	0.025
24	Israel	ILS	0.023	0.239	0	0.095
25	Italy	ITL	0.023	0.204	0	0.080
26	Japan	JPY	0.023	0.149	0	0.004
27	Kuwait	KWD	0.045	0.249	0.045	0.208
28	Malaysia	MYR	0.068	0.131	0.090	0.001
29	Mexico	MXN	0.045	0.174	0.045	0.014
30	New Zealand	NZD	0.045	0.204	0.045	0.037
31	Nigeria	NGN	0.023	0.206	0.000	0.081
32	Norway	NOK	0.091	0.312	0.539	0.311
33	Pakistan	PKR	0.023	0.173	0	0.027
34	Philippines	PHP	0.023	0.226	0	0.064
35	Polish	PLN	0.068	0.229	0.280	0.082
36	Russia	RUB	0.045	0.196	0.206	0.028
37	Saudi Arabia	SAR	0.023	0.148	0	0.004
38	Singapore	SGD	0.136	0.289	0.541	0.209
39	South Africa	ZAR	0.068	0.208	0.132	0.042
40	South Korea	KRW	0.023	0.116	0	0.000
41	Spain	ESP	0.023	0.204	0	0.080
42	Sweden	SEK	0.023	0.239	0	0.095
43	Swiss	CHF	0.023	0.246	0	0.189
44	Thailand	THB	0.045	0.173	0.045	0.013
45	Emirate	AED	0.023	0.200	0	0.064

To identify the 8 most important currencies based on all four centrality measures, we select the top 8 nodes for each centrality measure and calculate their cumulative distribution. Table 2 presents the frequency and level of each node across the centrality measures (degree, betweenness, closeness, and eigenvector centrality). These nodes are ranked by frequency from highest to lowest, with the 8 most significant currencies highlighted across all centrality measures (see Table 2).

Tab. 2. Frequency and level of 8 most important currencies

Node	Currency	Frequency	Level
FRF	French -francs	4	1
SGD	Singapore dollar	4	2,2,3,6
NOK	Norwegian kroner	4	6,3,2,2
XPF	CFP France	3	4,8,5,3
CZK	Czech koruna	3	5,10,6,4
GRD	Greece drachma	3	13,5,4,5
AUD	Australian dollar	2	3,4,7,12
PLN	Polish zloty	1	8

Table 2 indicates that FRF, SGD, and NOK rank at the highest levels in all four centrality measures, exhibiting the highest frequencies across the dataset. These currencies stand out as the strongest and most influential players in the currency network. The centrality measures highlight that these currencies have the most extensive connections with other currencies, allowing them to play a dominant role in the global financial network. As hubs in the currency market, these currencies not only control the flow of information but also facilitate the rapid spread of information across the network. Their ability to influence and respond to changes in global financial conditions makes them essential for any analysis of currency market dynamics. Investors should prioritize these currencies due to their ability to withstand market shocks and maintain stability, as they are more likely to remain resilient in times of financial turbulence.

Following these top currencies, XPF, CZK, GRD, AUD, and PLN occupy the second level of importance, with substantial influence in the network. Although they do not hold the same dominant position as FRF, SGD, and NOK, these currencies are still well-connected and have a significant impact on the currency market. Their connections enable them to affect global financial movements and absorb information from other parts of the market. These currencies are crucial for investors who aim to diversify their portfolios while maintaining exposure to key players in the currency network. Despite their second-tier status, their connectivity allows them to play an important role in shaping the broader economic landscape. However, their slightly lower centrality scores suggest they may be more susceptible to fluctuations in market sentiment or changes in economic conditions than the top-ranked currencies.

On the other hand, 10 currencies—ARS, KRW, SAR, JPY, CLP, BRL, PKR, BIF, IDR, and CAD—are found to be at the lowest levels according to the four centrality measures. These currencies show limited connections within the global network and tend to play a less influential role in the currency market. Their relative isolation makes them more vulnerable to domestic economic conditions, geopolitical events, and external shocks. These currencies are more likely to experience greater volatility, as they lack the robust connectivity necessary to absorb or disseminate market-moving information quickly. Investors should approach these currencies with caution, as their low centrality suggests they may face higher risks, including greater exposure to sudden market shifts and external pressures. While they may offer opportunities for high-risk, high-reward investments, they are also more prone to unpredictable fluctuations. Thus, investors should carefully assess the risks associated with these currencies and consider their portfolio’s overall risk tolerance when engaging with these markets.

Conclusion

In this paper, we measured the similarity among 2-dimensional currencies, where each currency is represented by a bivariate time series of bid and ask prices, to uncover hidden patterns in the currency market using the RV correlation coefficient. We applied this approach to analyze 45 major world currencies from May 2008 to May 2013. This research represents the first attempt to utilize Escoufier's vector correlation within a currency network, offering new insights into market behavior, risk propagation, and structural dependencies.

We conceptualized the complex system of bi-dimensional currencies as a network, enabling a more efficient understanding of the relationships among currencies. To extract key information from these networks, we employed the MST method. We then analyzed the statistical properties of the network using four centrality measures—degree, betweenness, closeness, and eigenvector centrality—and identified the eight most influential currencies based on the highest scores across all measures.

While previous research has primarily relied on the Pearson Correlation Coefficient (PCC) to measure the similarity of univariate time series of closing prices, an important contribution of this study is the integration of the RV coefficient method. This method captures the bi-dimensional information hidden in currency prices and is combined with the MST-based network approach to provide a deeper investigation into the statistical properties of the FOREX market.

Our analysis, utilizing both the RV coefficient and MST methods, offers valuable insights that can be applied to other financial markets, including the stock, commodity, and equity markets. Given the increasing frequency of financial crises, our work provides a framework for exploring the relationship between financial crises, currency crises, and the properties of the currency network in terms of multi-dimensional information. Furthermore, the RV coefficient method can be integrated with other network analysis techniques to study the topology of financial networks more comprehensively.

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